Chapter 6

Prerequisite Skills (p. 412)

1. The square roots of 81 are 9 and −9.
2. In the expression $2^5$, the exponent is 5.
3. For the polynomial function whose graph is shown, the sign of the leading coefficient is positive.

$$\frac{5x^2y}{15xy^{-1}} = \frac{5x^2 - 3y^{-1} - 1}{15}$$

4. $$\frac{5x^2 - 3y^{-1}}{15} = \frac{5x - 3y}{15} = \frac{x^2}{3x}$$

5. $$\frac{32x^{-3}y^4}{24x^{-3}y^{-2} \cdot 9y^2} = \frac{96x^{-3} + y^4}{216x^{-3}y^{-2} + 1}$$

6. $$(2x^5y^{-3})^{-3} = \frac{1}{(2x^5y^{-3})^3} = \frac{1}{2^3(x^5)^3(y^{-3})^3} = \frac{1}{8x^{15}y^{-9}} = \frac{y^9}{8x^{15}}$$

7. $$-2x - 5y = 10$$

$$-5y = 2x + 10$$

$$y = -\frac{2}{3}x - 2$$

8. $$x - \frac{1}{2}y = -1$$

$$3x - y = -3$$

$$y = 3x - 3$$

$$y = x + 3$$

9. $$8x - 4xy = 3$$

$$-4xy = 8x + 3$$

$$y = \frac{2}{3}x$$

10. $$f(x) = x^3 - 4x + 6$$

<table>
<thead>
<tr>
<th>x</th>
<th>-3</th>
<th>2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>-9</td>
<td>6</td>
<td>9</td>
<td>6</td>
<td>3</td>
<td>6</td>
<td>21</td>
</tr>
</tbody>
</table>

The degree is odd and the leading coefficient is positive, so $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to +\infty$ as $x \to +\infty$.

11. $$f(x) = -x^3 + 7x^2 + 2$$

<table>
<thead>
<tr>
<th>x</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>-62</td>
<td>10</td>
<td>2</td>
<td>8</td>
<td>-2</td>
</tr>
</tbody>
</table>

The degree is odd and the leading coefficient is negative, so $f(x) \to \infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to +\infty$.

12. $$f(x) = x^4 - 4x^2 + x$$

<table>
<thead>
<tr>
<th>x</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>42</td>
<td>-2</td>
<td>-4</td>
<td>0</td>
<td>-2</td>
<td>2</td>
<td>48</td>
</tr>
</tbody>
</table>

The degree is even and the leading coefficient is positive, so $f(x) \to +\infty$ as $x \to -\infty$ and $f(x) \to +\infty$ as $x \to +\infty$.

Lesson 6.1

6.1 Guided Practice (pp. 415–416)

1. Because $n = 4$ is even and $a = 625 > 0$, 625 has two real fourth roots. Because $5^4 = 625$ and $(-5)^4 = 625$, you can write $\pm \sqrt[4]{625} = \pm 5$ or $\pm 625^{1/4} = \pm 5$.

2. Because $n = 6$ is even and $a = 64 > 0$, 64 has two real 6th roots. Because $2^6 = 64$ and $(-2)^6 = 64$, you can write $\pm \sqrt[6]{64} = \pm 2$ or $\pm 64^{1/6} = \pm 2$.

3. Because $n = 3$ is odd and $a = -64 < 0$, -64 has one real cube root. Because $(-4)^3 = -64$, you can write $\sqrt[3]{-64} = -4$ or $(-64)^{1/3} = -4$.

4. Because $n = 5$ and $a = 243 > 0$, 243 has one real fifth root. Because $3^5 = 243$, you can write $\sqrt[5]{243} = 3$ or $243^{1/5} = 3$.

5. $4^{5/2} = (4^{2/2})^5 = 2^5 = 32$

6. $9^{1/2} = \frac{9^{1/2}}{3} = \frac{1}{3}$

7. $81^{3/4} = (\sqrt[4]{81})^3 = 3^3 = 27$

8. $17^{3/8} = (1^{1/8})^{3/8} = 1^{3} = 1$

9. $4^{2/5} = 1.74$

10. $64^{-2/3} = 0.06$

11. $(\sqrt[6]{16})^3 = 32$

12. $(\sqrt[3]{-30})^2 = 9.65$

13. $x^3 = 64$

$x = \sqrt[3]{64}$

$x = 4$

14. $\frac{1}{2}x^3 = 512$

$x^3 = 1024$

$x = \sqrt[3]{1024}$

$x = 4$
Chapter 6, continued

15. 3\(x^2\) = 108
   \(x^2 = 36\)
   \(x = \pm 6\)
16. \(\frac{3}{4}x^3 = 2\)
   \(x^3 = \frac{8}{3}\)
   \(x = \left(\frac{2}{3}\right)^{1/3} \approx 0.69\)
17. \((x - 2)^3 = -14\)
   \(x - 2 = \sqrt[3]{-14}\)
   \(x = \sqrt[3]{-14} + 2\)
   \(x = -0.41\)
18. \((x + 5)^4 = 16\)
   \(x + 5 = \sqrt[4]{16}\)
   \(x = \sqrt[4]{16} - 5\)
   \(x = -3\) or \(x = -7\)

19. \(w = 0.0167l^3\)
   a. \(275 = 0.0167l^3\)
      \(16,467 = l^3\)
      \(l = \sqrt[3]{16,467} \approx 25.4\)
      A coral cod that weighs 275 grams is about 25 centimeters long.
   b. \(340 = 0.0167l^3\)
      \(20,359 = l^3\)
      \(l = \sqrt[3]{20,359} \approx 27.3\)
      A coral cod that weighs 340 grams is about 27 centimeters long.
   c. \(450 = 0.0167l^3\)
      \(26,946 = l^3\)
      \(l = \sqrt[3]{26,946} \approx 30\)
      A coral cod that weighs 450 grams is about 30 centimeters long.

6.1 Exercises (pp. 417–419)

Skill Practice

1. In the expression \(\sqrt[4]{10,000}\), the number 4 is called the index of the radical.
2. If \(a < 0\), then there are no real fourth roots. If \(a > 0\), then there are two real fourth roots: \(\pm \sqrt[4]{a} = \pm a^{1/4}\). If \(a < 0\) or if \(a > 0\), then there is one real fifth root: \(\sqrt[5]{a} = a^{1/5}\).
3. C; \(21^{1/3} = \sqrt[3]{2}\)
4. A; \(2^{3/2} = (\sqrt{2})^3\)
5. D; \(2^{2/3} = (\sqrt[3]{2})^2\)
6. B; \(2^{1/2} = \sqrt{2}\)
7. \(\sqrt[3]{12} = 2^{1/3}\)
8. \(\sqrt[3]{8} = 2^{1/3}\)
9. \(\sqrt[3]{10} = 10^{1/3}\)
10. \(\sqrt[5]{15} = 15^{1/5}\)
11. \(5^{1/4} = \sqrt[4]{5}\)
12. \(7^{1/3} = \sqrt[3]{7}\)
13. \(14^{2/3} = (\sqrt[3]{14})^2\)

15. Because \(n = 2\) is even and \(a = 64 > 0\), 64 has two real square roots. Because \(8^2 = 64\) and \((-8)^2 = 64\), you can write \(\pm \sqrt[4]{64} = \pm 8\) or \(\pm 64^{1/2} = \pm 8\).
16. Because \(n = 3\) is odd and \(a = -27 < 0\), -27 has one real cube root. Because \((-3)^3 = -27\), you can write \(\sqrt[3]{-27} = -3\) or \((-27)^{1/3} = -3\).
17. Because \(n = 4\) is even and \(a = 0\), 0 has one real fourth root. Because \(0^4 = 0\), you can write \(\sqrt[4]{0} = 0\).
18. Because \(n = 3\) is odd and \(a = 343 > 0\), 343 has one real cube root. Because \(7^3 = 343\), you can write \(\sqrt[3]{343} = 7\) or \(343^{1/3} = 7\).
19. Because \(n = 4\) is even and \(a = -16 < 0\), -16 has no real fourth roots.
20. Because \(n = 5\) is odd and \(a = -32 < 0\), -32 has one real fifth root. Because \((-2)^5 = -32\), you can write \(\sqrt[5]{-32} = -2\) or \((-32)^{1/5} = -2\).
21. \(\sqrt[3]{64} = 2\)
22. \(8^{1/3} = 2\)
23. \(16^{1/2} = (16^{1/2})^3\)
24. \(\sqrt[3]{-125} = -5\)
   \(= 4^3 = 64\)
25. \(27^{1/3} = (27^{1/3})^2\)
26. \((-243)^{1/5} = -3\)
   \(= 3^2 = 9\)
27. \((\sqrt[3]{8})^2 = \left(\frac{1}{\sqrt[3]{8}}\right)^2 = 2\)
28. \((\sqrt[3]{-64})^4 = (-4)^4 = 256\)
29. \((\sqrt[5]{16})^7 = \left(\frac{1}{\sqrt[5]{16}}\right)^7 = \frac{1}{2^7} = \frac{1}{128}\)
30. \(25^{1/2} = (25^{1/2})^3\)
31. \(64^{-1/3} = \left(\frac{1}{64^{1/3}}\right)^3 = \frac{1}{4^3} = \frac{1}{16}\)
32. \(\frac{1}{81^{1/4}} = (81^{1/4})^3 = 3 = 27\)
33. C; \(128^{1/7} = (128^{1/7})^5 = 2^5 = 32\)
34. \(\sqrt[4]{32,768} = 8\)
35. \(\sqrt[4]{1695} \approx 2.89\)
36. \(\sqrt[3]{-250} = -1.83\)
37. \(85^{1/6} = 2.10\)
38. \(25^{1/3} = 0.34\)
39. \(20,736^{1/2} = 12\)
40. \(\sqrt[3]{817} = 50.57\)
41. \((\sqrt[5]{6})^5 = 0.01\)
42. \((\sqrt[3]{-8})^8 = 27.86\)
43. \(86^{-5/6} = 0.02\)
44. \(1974^{2/7} = 8.74\)
45. \(-1^{3/5} = (-17)^{-3/5} = -0.18\)
46. B; \(27^{1/5} = 7.22\)
47. Sample answer: \(729^{1/6}; 27^{1/3}\)
48. The cube root of 27 is 3, not 9.
   \(x^3 = 27\)
   \(x = \sqrt[3]{27}\)
   \(x = 3\)
49. The negative fourth root of 81 was not included.
   \(x^4 = 81\)
   \(x = \pm \sqrt[4]{81}\)
   \(x = \pm 3\)

Algebra 2

362 Worked-Out Solution Key
Chapter 6, continued

50. \( x^3 = 125 \)
   \( x = \sqrt[3]{125} \)
   \( x = 5 \)

51. \( 5x^3 = 1080 \)
   \( x^3 = 216 \)
   \( x = \sqrt[3]{216} \)
   \( x = 6 \)

52. \( x^8 + 36 = 100 \)
   \( x^8 = 64 \)
   \( x = \pm \sqrt[8]{64} \)
   \( x = \pm 2 \)

53. \( (x - 5)^4 = 256 \)
   \( x - 5 = \pm \sqrt[4]{256} \)
   \( x = \pm 5 + 5 \)
   \( x = 4 + 5 \) or \( x = -4 + 5 \)
   \( x = 9 \) or \( x = -1 \)

54. \( x^5 = -48 \)
   \( x = \sqrt[5]{-48} \)
   \( x = -2.17 \)

55. \( 7x^4 = 56 \)
   \( x^4 = 8 \)
   \( x = \pm \sqrt[4]{8} \)
   \( x = \pm 2 \)

56. \( x^3 + 40 = 25 \)
   \( x^3 = -15 \)
   \( x = \sqrt[3]{-15} \)
   \( x = -2.47 \)

57. \( (x + 10)^3 = 70 \)
   \( x + 10 = \sqrt[3]{70} \)
   \( x = -2.47 \)

58. \( x^6 - 34 = 181 \)
   \( x^6 = 215 \)
   \( x = \sqrt[6]{215} \)
   \( x = \pm 2.45 \)

59. a. When \( a < 0 \), you can see from the graph that the line \( y = a \) does not intersect the graph of \( y = x^n \). There are no real \( n \)th roots.
   When \( a = 0 \), you can see from the graph that the line \( y = a \) intersects the graph of \( y = x^n \) once. There is one real \( n \)th root.
   When \( a > 0 \), you can see from the graph that the line \( y = a \) intersects the graph of \( y = x^n \) twice. There are two real \( n \)th roots.

b.

\[
\begin{align*}
  y &= a \\
  y &= 0 \\
  y &= a \\
  a > 0 \quad a < 0
\end{align*}
\]

Problem Solving

60. \( V = \frac{4}{3} \pi r^3 \)
   \( 905 = \frac{4}{3} \pi r^3 \)
   \( 216.05 = r^3 \)
   \( \sqrt[3]{216.05} = r \)
   \( 6 = r \)

The radius of the shot is about 6 centimeters.

61. \( S = 4\pi r^2 \)
   \( 232 = 4\pi r^2 \)
   \( 18.46 = r^2 \)
   \( \pm \sqrt{18.46} = r \)
   \( \pm 4.3 = r \)

Reject the negative value, \(-4.3\). The radius of the bowling ball is about 4.3 inches.

62. \( r = \left( \frac{p_2}{p_1} \right)^{1/n} - 1 \)
   Butter: \( r = \left( \frac{2.195}{0.7420} \right)^{1/40} - 1 \)
   \( r = (2.96)^{1/40} - 1 \)
   \( r = 1.027 - 1 \)
   \( r = 0.027 = 2.7\% \)

   Chicken: \( r = \left( \frac{1.087}{0.4430} \right)^{1/40} - 1 \)
   \( r = (2.45)^{1/40} - 1 \)
   \( r = 1.023 - 1 \)
   \( r = 0.023 = 2.3\% \)

   Eggs: \( r = \left( \frac{1.356}{0.6710} \right)^{1/40} - 1 \)
   \( r = (2.02)^{1/40} - 1 \)
   \( r = 1.018 - 1 \)
   \( r = 0.018 = 1.8\% \)

   Sugar: \( r = \left( \frac{0.4560}{0.0936} \right)^{1/40} - 1 \)
   \( r = (4.87)^{1/40} - 1 \)
   \( r = 1.040 - 1 \)
   \( r = 0.040 = 4.0\% \)

63. \( p = ks^3 \)
   \( 1.2 = k(1700)^3 \)
   \( \frac{12}{4,094,166,667} = k \)

   When \( p = 1.5: 1.5 = \frac{1}{4,094,166,667}s^3 \)
   \( 6,141,250,000 = s^3 \)
   \( \sqrt[3]{6,141,250,000} = s \)
   \( 1831 = s \)

   The speed of the fan if it uses 1.5 horsepower is about 1831 revolutions per minute.

64. \( \bar{Q} = 3.367h^{1/2} \)
   \( \bar{Q} = 3.367(20)^{1/2} \)
   \( \bar{Q} = 753 \)

   The flow rate of the weir is about 753 cubic feet per second.
Chapter 6, continued

65. a. \( V = s^3 \)
    \( V = (16)^3 \)
    \( V = 4096 \)
    The volume of the cube is 4096 cubic millimeters.

b. Tetrahedron: \( V = 0.118x^3 \)
    \( 4096 = 0.118x^3 \)
    \( 34,712 = x^3 \)
    \( \sqrt[3]{34,712} = x \)
    \( 33 = x \)
    Edge length: about 33 millimeters

Octahedron: \( V = 0.471x^3 \)
    \( 4096 = 0.471x^3 \)
    \( 8696 = x^3 \)
    \( \sqrt[3]{8696} = x \)
    \( 21 = x \)
    Edge length: about 21 millimeters

Dodecahedron: \( V = 7.663x^3 \)
    \( 4096 = 7.663x^3 \)
    \( 535 = x^3 \)
    \( \sqrt[3]{535} = x \)
    \( 8 = x \)
    Edge length: about 8 millimeters

Icosahedron: \( V = 2.182x^3 \)
    \( 4096 = 2.182x^3 \)
    \( 1877 = x^3 \)
    \( \sqrt[3]{1877} = x \)
    \( 12 = x \)
    Edge length: about 12 millimeters

e. No; the icosahedron has the greatest number of faces but not the smallest edge length. Of the three polyhedra with equilateral triangles for faces, the icosahedron has the smallest edge length, but the dodecahedron’s faces are regular pentagons, so its edge length is smaller.

66. An equation for mass \( m \) in terms of speed \( s \) is \( m = ks^6 \), where \( k \) is a constant.

At speed of 1 meter per second: \( m = k(1)^6 = k \)

Twice as massive: \( 2k = ks^6 \)
    \( 2 = s^6 \)
    \( s = \sqrt[6]{2} \)
    \( \pm 1.1 = s \)

Reject the negative value, \(-1.1\). The river’s speed must be about 1.1 meters per second to transport particles that are twice as massive as usual.

10 times as massive: \( 10k = ks^6 \)
    \( 10 = s^6 \)
    \( s = \sqrt[6]{10} \)
    \( \pm 1.5 = s \)

Reject the negative value, \(-1.5\). The river’s speed must be about 1.5 meters per second to transport particles that are 10 times as massive as usual.

100 times as massive: \( 100k = ks^6 \)
    \( 100 = s^6 \)
    \( \pm \sqrt[6]{100} = s \)
    \( \pm 2.2 = s \)

Mixed Review

67. \( x = 3, y = 5 \)
    \( x + 3y = 3 + 3(5) \)
    \( x - y = 3 - 3 \)
    \( x - 2y = 3 - 2(5) \)
    \( x = 18 \)
    \( -2 = -9 \)
    \( 26 = 13 \)

68. \( x = 6, y = -2 \)
    \( 4x - y = 4(6) - (-2) \)
    \( x - 2y = 6 - 2(-2) \)
    \( x = 26 \)
    \( 10 = 13 \)

69. \( f(x) = x^2 - 2x - 35 = (x - 7)(x + 5) \)

The zeros are 7 and -5.

70. \( f(x) = x^2 - 8x + 25 \)
    \( x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(25)}}{2(1)} \)
    \( x = \frac{8 \pm \sqrt{36}}{2} \)
    \( x = \frac{8 \pm 6i}{2} \)
    \( x = 4 \pm 3i \)

The zeros are \( 4 + 3i \) and \( 4 - 3i \).

71. \( f(x) = x^3 - 8x^2 + 4x - 32 \)
    \( x = \pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm 32 \)

Test \( x = 8 \):

<table>
<thead>
<tr>
<th></th>
<th>8</th>
<th>-8</th>
<th>4</th>
<th>-32</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

\( f(x) = (x - 8)(x^2 + 4) \)

\( x^2 + 4 = 0 \)
    \( x^2 = -4 \)
    \( x = \pm 2i \)

\( f(x) = (x - 8)(x - 2i)(x + 2i) \)

The zeros are 8, 2i, and -2i.

72. \( f(x) = x^3 - 4x^2 + 25x + 100 \)
    \( x = \pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20, \pm 25, \pm 50, \pm 100 \)

Test \( x = -4 \):

<table>
<thead>
<tr>
<th></th>
<th>-4</th>
<th>1</th>
<th>4</th>
<th>25</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>-4</td>
<td>0</td>
<td>-100</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( f(x) = (x + 4)(x^2 + 25) = (x + 4)(x - 5i)(x + 5i) \)

The zeros are -4, 5i, and -5i.
Chapter 6, continued

73. \( f(x) = x^4 - 3x^3 - 31x^2 + 63x + 90 \)
   \[ x = \pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 9, \pm 10, \pm 15, \pm 18, \pm 30, \pm 45 \]
   Test \( x = 3 \):
   \[
   \begin{array}{c|cccc}
   3 & -3 & -31 & 63 & 90 \\
   1 & 0 & -31 & -30 & 0 \\
   \end{array}
   \]
   \( f(x) = (x - 3)(x^3 - 31x - 30) \)
   \[ x = \pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30 \]
   Test \( x = -5 \):
   \[
   \begin{array}{c|cccc}
   -5 & 1 & 0 & -31 & -30 \\
   1 & -5 & -31 & 25 & 30 \\
   \end{array}
   \]
   \( f(x) = (x - 3)(x + 5)(x^2 - 5x - 6) \)

74. \( f(x) = x^4 + 10x^3 + 25x^2 - 36 \)
   \[ x = \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9, \pm 12, \pm 18, \pm 36 \]
   Test \( x = 1 \):
   \[
   \begin{array}{c|cccc}
   1 & 10 & 25 & 0 & -36 \\
   1 & 11 & 36 & 36 & 0 \\
   \end{array}
   \]
   \( f(x) = (x - 1)(x^3 + 11x^2 + 36x + 36) \)

75. \( \frac{x^4}{x^3} = x^{4-3} = x^1 \)
   \[ = \frac{1}{x} \]
   Quotient of powers property

76. \( (x^3)^{-2} = x^{-6} \)
   \[ = \frac{1}{x^6} \]
   Power of a power property

77. \( (3xy^2)^{-3} = \frac{1}{(3xy^2)^3} \)
   \[ = \frac{1}{27x^3y^6} \]
   Negative exponent property

78. \( 4x^0y^{-4} = 4y^{-4} \)
   \[ = \frac{4}{y^4} \]
   Zero exponent property

79. \( x^6 \cdot x^{-2} = x^{6-(-2)} \)
   \[ = x^8 \]
   Product of powers property

80. \( \frac{x^3}{y^{-2}} \cdot \frac{y^2}{x^{-2}} = \frac{(x^3)(y^2)}{(x^{-2})(y^{-2})} \)
   \[ = \frac{x^6}{y^0} = x^6y^4 \]
   Power of a quotient property

81. \( \frac{4x^3y^6}{10xy^{-3}} = \frac{4x^3y^6}{10x} \)
   \[ = \frac{2x^2y^9}{5} \]
   Quotient of powers property

82. \( \frac{3x}{xy^2} \cdot \frac{y^4}{9x^2} \)
   \[ = \frac{3xy^4}{9x^3y^2} \]
   Quotient of powers property
   \[ = \frac{3y^2}{9} \]
   Product of powers property
   \[ = \frac{3y^2}{3} \]
   Quotient of powers property

Lesson 6.2

6.2 Guided Practice (pp. 421–423)

1. \( (5^{1/3} \cdot 7^{1/4})^3 = (5^{1/3})^3 \cdot (7^{1/4})^3 \)
   \[ = 5^{3/3} \cdot 7^{3/4} \]
   \[ = 5 \cdot 7^{3/4} \]

2. \( 2^{3/4} \cdot 2^{1/2} = 2^{(3/4) + (1/2)} = 2^{5/4} \)

3. \( \frac{3}{\sqrt[3]{4}} = \frac{3^{1/3}}{4^{1/3}} = \frac{3(1 - 1/4)}{3^{1/3}} = 3^{1/3} \)

4. \( \left( \frac{20^{1/2}}{5^{1/2}} \right)^3 = \left( \frac{20}{5} \right)^{3/2} = (4^{1/2})^3 = 2^3 = 8 \)

5. \( S = \frac{km}{23} \)
   \[ = 8.4(9.5 \times 10^5)^{2/3} \]
   \[ = 8.4(9.5)^{2/3}(10^5)^{2/3} \]
   \[ = 8.4(4.49)(10^{16/3}) \]
   \[ = 17,506 \]

The sheep’s surface area is about 17,506 square centimeters.

6. \( \sqrt[3]{27} \cdot \sqrt[3]{3} = \sqrt[3]{27 \cdot 3} = \sqrt[3]{81} = 3 \)

7. \( \sqrt{250} = \sqrt[2]{250} = \sqrt{125} = 5 \)

8. \( \sqrt[4]{\frac{3}{4}} \cdot \sqrt[8]{\frac{8}{8}} = \sqrt[8]{\frac{3}{4} \cdot 8} = \sqrt[8]{24} = \sqrt[2]{24} = 2 \)
9. \( \sqrt{5} + \sqrt{40} = \sqrt{5} + \sqrt{8 \cdot 5} = \sqrt{5} + 2\sqrt{5} = 3\sqrt{5} \)
10. \( \sqrt[3]{27}q^3 = \sqrt[3]{3^3 \cdot q^3} = 3q^3 \)
11. \( \sqrt[10]{8} = \sqrt[10]{(2^3)^{\frac{1}{10}}} = \sqrt[10]{2^3} = \frac{\sqrt[10]{2}}{\sqrt[10]{y}} \)
\( \frac{2x^4 - 1}{2x^3 - 1} = 2x^{1/2} \left( \frac{3x^3 - 1}{2x^3 - 1} \right) = 2x^{1/2} \left( \frac{1}{2} \right) \)
12. \( \sqrt[3]{9w^5} - \sqrt[3]{w^3} = 3w^2\sqrt[3]{w} - w^2\sqrt[3]{w} = (3w^2 - w^2)\sqrt[3]{w} = 2w^2\sqrt[3]{w} \)

6.2 Exercises (pp. 424–427)

Skill Practice

1. No, \( 2\sqrt{5} \) and \( \sqrt{5} \) are not like radicals because they do not have the same index. The expression \( 2\sqrt{5} \) has an index of 2 and \( \sqrt{5} \) has an index of 3.
2. A radical expression with index \( n \) is in simplest form if the radicand has no perfect \( n \)th powers as factors and any denominator has been rationalized.
3. \( 5^{3/2} \cdot 5^{1/2} = 5^{3/2 + 1/2} = 5^2 = 25 \)
4. \( (6^{2/3})^{1/2} = 6^{2/3 \cdot 1/2} = 6^{1/3} \)
5. \( 3^{1/4} 
\cdot 2^{7/14} = (3 \cdot 2^{7})^{1/4} \) = \( 81^{1/4} = 3 \)
6. \( \frac{9}{9 - 45} = 9^{1/4} = 9^{9/9} \)
7. \( \frac{80/14}{5 - 12} = \frac{80^{1/4} \cdot 5^{1/4}}{(80 - 5)^{1/4}} = \left( \frac{80 \cdot 5}{75} \right)^{1/4} = \frac{400^{1/4}}{11^{1/4}} = 16^{1/4} \cdot 25^{1/4} = 2 \cdot 5^{1/2} \)
8. \( \left( \frac{n}{4} \right)^{1/3} = \left( \frac{3}{4} \right)^{1/3} = \left( \frac{3^{1/3}}{4^{1/3}} \right) = \left( \frac{\sqrt[3]{3}}{\sqrt[3]{4}} \right) = \frac{\sqrt[3]{3}}{\sqrt[3]{4}} = \frac{3}{4} \)
9. \( \frac{11^{2/5}}{11^{1/5}} = 11^{2/5 - 1/5} = 11^{-1/5} = \frac{1}{\sqrt[5]{11}} = \sqrt[5]{\frac{1}{11}} \)
10. \( \sqrt[8]{(12 \cdot 8)^{3/5}} = \sqrt[8]{96^{3/5}} = \sqrt[8]{960} = \sqrt[8]{884,736} \)
11. \( \sqrt[7]{120 - 2^{3/5} \cdot 120^{2/5}} = \sqrt[7]{7^{3/4} - 7^{2/3} \cdot 7^{1/2}} = 7^{3/4} \)
12. \( \sqrt[4]{64^{1/2} \cdot 64^{1/2}} = \sqrt[4]{(64^{1/2})^2} = \sqrt[4]{64^{1/2 \cdot 2}} = \sqrt[4]{64^{3/4}} = 4 \)

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26. \( \sqrt[3]{108} \cdot \sqrt[4]{4} = \sqrt[4]{108} \cdot 4 \)
   \( = \sqrt[4]{432} \)
   \( = \sqrt[3]{216} \cdot 2 \)
   \( = \sqrt[3]{216} \cdot \sqrt[3]{2} \)
   \( = 6 \sqrt[3]{2} \)

27. \( 5 \sqrt[5]{64} \cdot 2 \sqrt[8]{8} = 5 \cdot 2 \sqrt[5]{64} \cdot 8 \)
   \( = 10 \sqrt[5]{512} \)
   \( = 10 \sqrt[5]{256} \cdot 2 \)
   \( = 10 \sqrt[5]{256} \cdot \sqrt[10]{2} \)
   \( = 10 \cdot 4 \cdot \sqrt[10]{2} \)
   \( = 40 \sqrt[10]{2} \)

28. \( \sqrt[3]{\frac{1}{6}} = \frac{1}{\sqrt[3]{6}} \)
   \( = \frac{\sqrt[3]{6}}{\sqrt[3]{6} \cdot 6} \)
   \( = \frac{\sqrt[3]{6}}{6} \)

29. \( \frac{3}{\sqrt[4]{144}} = \frac{3}{\sqrt[4]{16} \cdot 9} \)
   \( = \frac{3}{\sqrt[4]{16} \cdot \sqrt[4]{9}} \)
   \( = \frac{3}{2 \sqrt[4]{9} \cdot \sqrt[4]{9}} \)
   \( = \frac{3}{2 \sqrt[4]{81}} \)
   \( = \frac{3}{2 \cdot 3} \)
   \( = \frac{1}{2} \)

30. \( \sqrt[3]{\frac{81}{4}} = \frac{\sqrt[3]{81}}{\sqrt[3]{4}} = \frac{\sqrt[3]{81}}{\sqrt[3]{4}} \)
   \( = \frac{\sqrt[3]{81}}{\sqrt[3]{16}} = \frac{\sqrt[3]{81}}{\sqrt[3]{16}} \)
   \( = \frac{\sqrt[3]{81}}{\sqrt[3]{16}} \)
   \( = \frac{\sqrt[3]{81}}{\sqrt[3]{16}} \)

31. \( \sqrt[6]{9} \cdot \sqrt[9]{9} = \sqrt[576]{9^{13} \cdot 9^{15}} \)
   \( = \sqrt[576]{9^{13} + 15} \)
   \( = \sqrt[576]{9^{13} + 15} \)
   \( = \sqrt[576]{9^{13} + 15} \)
   \( = \sqrt[576]{\frac{9^{13} + 15}{3}} \)
   \( = \frac{9^{13} + 15}{3} \)
   \( = \frac{9^{13} + 15}{3} \)

32. \( 2 \sqrt[3]{3} + 7 \sqrt[3]{3} = (2 + 7) \sqrt[3]{3} = 9 \sqrt[3]{3} \)

33. \( \frac{3}{\sqrt[5]{5}} \cdot \frac{3}{\sqrt[5]{5}} = \frac{3}{\sqrt[5]{5}} \cdot \frac{3}{\sqrt[5]{5}} \)
   \( = \frac{3}{\sqrt[5]{5}} \cdot \frac{3}{\sqrt[5]{5}} \)
   \( = \frac{3}{\sqrt[5]{5}} \cdot \frac{3}{\sqrt[5]{5}} \)
   \( = \frac{3}{\sqrt[5]{5}} \cdot \frac{3}{\sqrt[5]{5}} \)

34. \( 25 \sqrt[2]{2} - 15 \sqrt[2]{2} = (25 - 15) \sqrt[2]{2} = 10 \sqrt[2]{2} \)

35. \( \frac{1}{\sqrt[3]{7}} + \frac{3}{\sqrt[3]{7}} = \frac{1}{\sqrt[3]{7}} + \frac{3}{\sqrt[3]{7}} = \frac{\sqrt[3]{7}}{2} \)

36. \( 6 \sqrt[2]{5} + 4 \sqrt[2]{625} = 6 \sqrt[2]{5} + 4 \sqrt[2]{125} \cdot 5 \)
   \( = 6 \sqrt[2]{5} + 4 \sqrt[2]{125} \cdot \sqrt[2]{5} \)
   \( = 6 \sqrt[2]{5} + 4 \cdot 5 \sqrt[2]{5} \)
   \( = 6 \sqrt[2]{5} + 20 \sqrt[2]{5} \)
   \( = (6 + 20) \sqrt[2]{5} \)
   \( = 26 \sqrt[2]{5} \)

37. \( -6 \sqrt[2]{2} + 2 \sqrt[2]{256} = -6 \sqrt[2]{2} + 2 \sqrt[2]{128} \cdot 2 \)
   \( = -6 \sqrt[2]{2} + 2 \sqrt[2]{128} \cdot \sqrt[2]{2} \)
   \( = -6 \sqrt[2]{2} + 2 \cdot 2 \cdot \sqrt[2]{2} \)
   \( = -6 \sqrt[2]{2} + 4 \sqrt[2]{2} \)
   \( = (-6 + 4) \sqrt[2]{2} = -2 \sqrt[2]{2} \)

38. \( 12 \sqrt[2]{2} - 7 \sqrt[2]{512} = 12 \sqrt[2]{2} - 7 \sqrt[2]{256} \cdot 2 \)
   \( = 12 \sqrt[2]{2} - 7 \sqrt[2]{256} \cdot \sqrt[2]{2} \)
   \( = 12 \sqrt[2]{2} - 7 \cdot 4 \sqrt[2]{2} \)
   \( = 12 \sqrt[2]{2} - 28 \sqrt[2]{2} \)
   \( = (12 - 28) \sqrt[2]{2} \)
   \( = -16 \sqrt[2]{2} \)

39. \( 2 \sqrt[2]{1250} - 8 \sqrt[2]{16} = 2 \sqrt[2]{125} \cdot 2 - 8 \sqrt[2]{16} \cdot 2 \)
   \( = 2 \sqrt[2]{125} \cdot 2 - 8 \sqrt[2]{16} \cdot \sqrt[2]{2} \)
   \( = 2 \cdot 5 \sqrt[2]{2} - 8 \cdot 2 \sqrt[2]{2} \)
   \( = 10 \sqrt[2]{2} - 16 \sqrt[2]{2} \)
   \( = (10 - 16) \sqrt[2]{2} \)
   \( = -6 \sqrt[2]{2} \)

40. \( 5 \sqrt[4]{48} - 7 \sqrt[4]{750} = 5 \sqrt[4]{8} \cdot 6 - \sqrt[4]{125} \cdot 6 \)
   \( = 5 \sqrt[4]{8} \cdot 6 - \sqrt[4]{125} \cdot \sqrt[4]{6} \)
   \( = 5 \cdot 2 \cdot 6 - 5 \cdot \sqrt[4]{6} \)
   \( = 10 \sqrt[4]{6} - 5 \sqrt[4]{6} \)
   \( = (10 - 5) \sqrt[4]{6} \)
   \( = 5 \sqrt[4]{6} \)

41. The radical expressions \( 2 \sqrt[2]{10} \) and \( 6 \sqrt[2]{5} \) are not like radicals because they don’t have the same radicand. Therefore, \( 2 \sqrt[2]{10} + 6 \sqrt[2]{5} \) cannot be combined.

42. To make the denominator a perfect cube, you must multiply the numerator and denominator of the fraction by \( y \) so that the value of the fraction does not change.
   \( \frac{\sqrt[3]{x} \cdot y}{\sqrt[3]{y^2} \cdot y} = \frac{\sqrt[3]{xy}}{\sqrt[3]{y^3}} = \sqrt[3]{\frac{xy}{y^3}} \)

43. \( x^{1/4} \cdot x^{1/3} = x^{1+1/3} = x^{4/3} = x^{4/3} \)

44. \( (y^2)^{1/6} = y^{2 \cdot 1/6} = y^{2/6} \)

45. \( \sqrt[3]{81x} = 3 \cdot x^{1/3} = \sqrt[3]{3x} \cdot \sqrt[3]{x^2} = 3x \)

46. \( \frac{2}{x^{3/2}} = 2x^{1/2} \)

47. \( \frac{2 \sqrt[3]{y}}{x^{1/3}} = x^{2/3 - 1/3} = x^{3/3} = x^{4/3} \)

48. \( \sqrt[3]{x^{15}} = \sqrt[3]{x^{15}} = \sqrt[3]{x^{27}} = x^{27} \)

49. \( (\sqrt[3]{x^2} \cdot \sqrt[4]{x^4})^3 = \frac{1}{(x^{2/3} \cdot x^{4/6})^3} = \frac{1}{(x^{2/3} \cdot 4x^6)^3} = \frac{1}{(x^{2/3} \cdot 4x^6)^3} = \frac{1}{x^2} \)
Chapter 6, continued

50. \[ \frac{\sqrt{5} \cdot \sqrt{2^3}}{\sqrt{25} \cdot \sqrt{2^6}} = \frac{\sqrt{5} \cdot \sqrt{2^3}}{25 \cdot \sqrt{2}^2} = \frac{\sqrt{5} \cdot 2^3}{5 \cdot 2^2} = \frac{x^{1/2} \cdot x}{5 \cdot x} = \frac{x^{1/2}}{x} = 5^{1/2} \]

51. Sample answer: \( x^{7/8}, x^{7/8} \)

52. \( \sqrt[4]{9x^4} = \sqrt[4]{9} \cdot x^4 = \sqrt[4]{9} \cdot x \cdot x = 3x \cdot \sqrt[4]{x} \)

53. \( \sqrt[6]{12} \cdot \sqrt[6]{2^2} = \sqrt[6]{12 \cdot 2^2} = \sqrt[6]{12 \cdot 4} = \sqrt[6]{48} \cdot \sqrt[6]{7} \)

54. \( \sqrt[4]{x^3} \cdot \sqrt[4]{x} = \sqrt[4]{x^3 \cdot x} = \sqrt[4]{x^4} = x \)

55. \( \sqrt{x^4} \cdot x^2 = \sqrt{x^6} = x^3 \cdot \sqrt{x} \)

56. \( -3 \cdot \sqrt[5]{x^2} \cdot y^3 = -3 \cdot \sqrt[5]{x^2} \cdot y^3 = -\sqrt[5]{x^2} \cdot y^3 \)

57. \( \frac{\sqrt[3]{y^2}}{\sqrt[3]{y^4}} = \frac{\sqrt[3]{y^2}}{\sqrt[3]{y^4}} \cdot \frac{\sqrt[3]{y^2}}{\sqrt[3]{y^2}} \cdot \frac{\sqrt[3]{y^2}}{\sqrt[3]{y^2}} = \sqrt[3]{x} \cdot \sqrt[3]{x} \cdot \sqrt[3]{x} = x \cdot x \cdot x = x^3 \)

58. \( \frac{20x^3y^2}{9x^3} = \frac{20x^3y^2}{9x^3} \cdot \frac{9x^3}{9x^3} = \frac{20}{9} \cdot \frac{x^3y^2}{x^3} = \frac{20}{9} \cdot x^3y^2 = \frac{20}{9} \cdot x^3y^2 \)

59. \( \frac{\sqrt[6]{x^2}}{\sqrt[6]{x}} = \frac{\sqrt[6]{x^2}}{\sqrt[6]{x}} \cdot \frac{\sqrt[6]{x^2}}{\sqrt[6]{x^2}} \cdot \frac{\sqrt[6]{x^2}}{\sqrt[6]{x^2}} = \frac{\sqrt[6]{x^2}}{\sqrt[6]{x^2}} \cdot \frac{\sqrt[6]{x^2}}{\sqrt[6]{x^2}} \cdot \frac{\sqrt[6]{x^2}}{\sqrt[6]{x^2}} = \frac{x^{2/3}}{x^{1/3}} = x^{1/3} \)

60. \( 3 \sqrt[6]{x} + 9 \sqrt[6]{x} = (3 + 9) \sqrt[6]{x} = 12 \sqrt[6]{x} \)

61. \( \frac{3}{4} x^{3/2} = \frac{3}{4} \cdot -\frac{1}{2} = \frac{3}{4} \cdot -\frac{1}{2} = -\frac{3}{2} \)

62. \( -7 \sqrt[6]{y^5} + 16 \sqrt[6]{y^5} = (-7 + 16) \sqrt[6]{y^5} = 9 \sqrt[6]{y^5} \)

63. \( (x^3y^2)^{1/2} + (xy^{1/2})^2 = x^{3/2} \cdot y^{2/2} + x^{1/2} \cdot y^{1/2} + x^{1/2} \cdot y^{1/2} = x^{3/2} + y^{1/2} + x^{1/2} \)

64. \( x \sqrt[6]{9x^2} - 2 \sqrt[6]{x} = x \sqrt[6]{9x^2} - 2 \sqrt[6]{x} = 3x \sqrt[6]{x} - 2x \sqrt[6]{x} = (3 - 2) \sqrt[6]{x} = x \sqrt[6]{x} \)

65. \( y \sqrt[6]{32x^6} + y \sqrt[6]{25x^3} = y \sqrt[6]{16 \cdot 2x^3} + y \sqrt[6]{25 \cdot 2x^3} = 2xy \sqrt[6]{2x^3} + 3y \sqrt[6]{2x^3} = (2xy + 3y \sqrt[6]{2x^3} \)

66. \( P = 2t + 2w \)

67. \( P = 2t + 2w \)

68. \( c^2 = a^2 + b^2 \)

69. \( C; \frac{1}{6} \sqrt[6]{4x} = \frac{1}{6} \sqrt[6]{9x} = \frac{1}{6} \cdot 2 \sqrt[6]{x} = \frac{1}{6} \cdot 3 \sqrt[6]{x} \)

70. \( x^{0.5} \cdot x^2 = x^{0.5 + 2} = x^{2.5} \)

71. \( y^{-0.6} \cdot y^{-6} = y^{-0.6 + (-6)} = y^{-6.6} = \frac{1}{y^{6.6}} \)

72. \( (x^2)^{0.75} = x^{2 \cdot (0.75)} = x^{1.5} = x^2 \cdot x^{1.5} = \frac{1}{x^{1.5}} \)

73. \( x^{0.3} = x^{(0.3 - 1.5)} = x^{-1.2} = \frac{1}{x^{1.2}} \)

74. \( (x^2y^3)^{-0.25} = x^{2 \cdot (-0.25)} y^{3 \cdot (-0.25)} = x^{-0.5} y^{-0.75} = \frac{1}{x^{0.5} y^{0.75}} \)

75. \( y^{0.5} = y^{(0.5 - 0.8)} = y^{-0.3} = \frac{1}{y^{0.3}} \)

76. \( 10^{0.6} + (4x^{0.3})^2 = 10^{0.6} + 16x^{0.6} \cdot 2 = 10^{0.6} + 16x^{0.6} = 26x^{0.6} \)
Chapter 6, continued

77. \(15^2.0^3 - (2^0.1)^3 = 15^2.0^3 - 8^2.0^3 = 72.0^3\)

78. \(\frac{x^{2/3}}{x^{5/3}} = x^{(2/3 - 5/3)} = x^{-3/3}\)

79. \((x^{1/2})^{3} = x^{(1/2 \times 3)} = x^{3/2}\)

80. \(\frac{3^2}{x^{2/3}} = x^{(-2/3)} = x^{(-2 \times -3/2)} = x^{4/3}\)

81. \(x^{2/3} + 3x^{2/3} = 4x^{2/3}\)

82. a. \(\frac{3}{9^2} = 243\)  

b. \(2^3 \cdot 2^x + 1 = \frac{1}{16}\)  

\[3 = 243 \cdot 9^x\]  

\[2^x + 1 = \frac{1}{16}\]  

\[\frac{3}{243} = 9^x\]  

\[\frac{3}{9^2} = 9^x\]  

\[\frac{2^x + 1}{16} = \frac{1}{16}\]  

\[\frac{1}{81} = 9^x\]  

\[\frac{1}{9^2} = 9^x\]  

\[2^x = 1 = 2\]  

\[9^{x^2} = 9^x\]  

\[2x + 1 = -4\]  

\[2x = -5\]  

\[x = -\frac{5}{2}\]  

x = -3 or \(x = 1\)

c. \((4^3)^2 + 2 = 64\)

\[4^3 \cdot (x + 2) = 64\]

\[4^2 + 2x = 64\]

\[4^2 + 2x = 4^2\]

\[x^2 + 2x = 3\]

\[x^2 + 2x - 3 = 0\]

\[(x + 3)(x - 1) = 0\]

\[x = -3\] or \(x = 1\)

Problem Solving

83. \(S = \frac{4}{3} \pi r^3\)

\[\frac{3V}{4\pi} = r^3\]

\[\sqrt[3]{\frac{3V}{4\pi}} = r\]

\[\frac{3}{4\pi} = r\]

b. \(S = 4\pi r^2\)

\[4\pi \left(\frac{3V}{4\pi}\right)^{1/2}\]

\[4\pi \left(\frac{3V}{4\pi}\right)^{1/2}\]

\[4\pi \left(\frac{3V}{4\pi}\right)^{1/2}\]

\[4\pi \left(\frac{3V}{4\pi}\right)^{1/2}\]

\[4\pi \left(\frac{3V}{4\pi}\right)^{1/2}\]

c. Balloon with volume \(V:\n
\[S_1 = (4\pi)^{1/3}(3V)^{2/3}\]

Balloon with volume \(2V:\n
\[S_2 = (4\pi)^{1/3}(3V)^{2/3}\]

\[= (4\pi)^{1/3}(3V)^{2/3}\]

\[= (4\pi)^{1/3}(3V)^{2/3}\]

\[= (4\pi)^{1/3}(3V)^{2/3}\]

\[= \sqrt{4}(4\pi)^{1/3}(3V)^{2/3}\]

\[= \sqrt[3]{4}(4\pi)^{1/3}(3V)^{2/3}\]

\[= \sqrt[3]{4}(4\pi)^{1/3}(3V)^{2/3}\]

The balloon with twice as much water has \(\sqrt[3]{4},\) or about 1.59 times the surface area of the other balloon.

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90. Sample answer: $\sqrt[4]{x^m}$
   \[ n = 3, m = 5, x = -1: \sqrt[4]{(-1)^5} = -1 \]
   \[ n = 4, m = 5, x = -1: \sqrt[4]{(-1)^5} = \sqrt[4]{-1}, \text{ but } -1 \text{ has no} \]
   \[ \text{real 4th root; } \sqrt[4]{(-1)^5} = 1 \]
   \[ n = 3, m = 4, x = -1: \sqrt[3]{(-1)^4} = 1 \]
   Absolute value is needed when $n$ is even and $m$ is odd.

Mixed Review

91. $x - 7 \geq 15$
   \[ x \geq 22 \]
92. $10x + 7 < -4x + 9$
   \[ 14x + 7 < 9 \]
   \[ 14x < 2 \]
   \[ x < \frac{1}{7} \]
93. $3x \leq -6x - 20$
   \[ 9x \leq -20 \]
   \[ x \leq -\frac{20}{9} \]
94. $x^2 + 7x + 10 > 0$
   \[ x^2 + 7x + 10 = 0 \]
   \[ (x + 5)(x + 2) = 0 \]
   \[ x = -5 \text{ or } x = -2 \]
   \[ \begin{array}{c|c}
   x < -5 & \text{ or } x > -2 \\
   \hline
   -6 & -5 \\
   -5 & -4 \\
   -3 & -2 \\
   -1 & 0 \\
   1 & 2 \\
   \end{array} \]

95. $-x^2 + 4x \geq -32$
   \[ 0 \geq x^2 - 4x - 32 \]
   \[ 0 = (x - 8)(x + 4) \]
   \[ x = 8 \text{ or } x = -4 \]
   \[ \begin{array}{c|c}
   x < -4 & \text{ or } x > 8 \\
   \hline
   -5 & -4 \\
   -4 & -3 \\
   -3 & -2 \\
   -2 & -1 \\
   -1 & 0 \\
   1 & 2 \\
   \end{array} \]

96. $6x^2 + x - 7 < 5$
   \[ 6x^2 + x - 12 < 0 \]
   \[ 6x^2 + x - 12 = 0 \]
   \[ (3x - 4)(2x + 3) = 0 \]
   \[ x = \frac{4}{3} \text{ or } x = -\frac{3}{2} \]
   \[ \begin{array}{c|c}
   x < -\frac{3}{2} & \text{ or } x > \frac{4}{3} \\
   \hline
   -3 & -2 \\
   -2 & -1 \\
   -1 & 0 \\
   0 & 1 \\
   1 & 2 \\
   2 & 3 \\
   \end{array} \]

97. \[
\begin{array}{cccc}
3 & 1 & -2 & -1 \\
3 & 3 & 6 \\
1 & 1 & 2 & 3 \\
\end{array}
\]
98. \[
\begin{array}{cccc}
-3 & 1 & -2 & -1 \\
-3 & 15 & -42 \\
1 & -5 & 14 & -45 \\
\end{array}
\]
   \[ f(3) = 3 \]
   \[ f(-3) = -45 \]

99. \[
\begin{array}{cccc}
5 & 1 & -2 & -1 \\
5 & 15 & 70 \\
1 & 3 & 14 & 67 \\
\end{array}
\]
   \[ f(5) = 67 \]

100. \[
\begin{array}{cccc}
-4 & 1 & -2 & -1 \\
-4 & 24 & -92 \\
1 & -6 & 23 & -95 \\
\end{array}
\]
   \[ f(-4) = -95 \]

101. \[(12x^2 + 2x) - (-8x^2 + 5x^2 - 9x) = 12x^2 + 2x + 8x^3 - 5x^2 + 9x = 8x^3 + 7x^2 + 11x \]
   \[ (35x^3 - 14) + (-15x^2 + 7x + 20) = 35x^3 - 15x^2 + 7x - 14 + 20 = 35x^3 - 15x^2 + 7x + 6 \]
   \[ 18x^2(x + 4) = 18x^2(x) + 18x^2(4) = 18x^3 + 72x^2 \]
   \[ (8x - 3)^2 = (8x)^2 - 2(8x)(3) + (-3)^2 = 64x^2 - 48x + 9 \]
   \[ (x - 4)(x + 1)(x + 2) = (x^2 - 3x - 4)(x + 2) = (x^2 - 3x - 4)x + (x^2 - 3x - 4)2 = x^3 - 3x^2 - 4x + 2x^2 - 6x - 8 = x^3 - 3x^2 - 10x - 8 \]
   \[ 106. \begin{array}{ccc}
3 & 1 & 1 \\
3 & 12 & 15 \\
1 & 4 & 5 \\
0 & 3 & 2 \\
\hline
\end{array} \]
   \[ x^3 + x^2 - 7x - 15 \]
   \[ x - 3 \]
   \[ x^3 + x^2 - 7x - 15 = x^2 + 4x + 5 \]
Chapter 6, continued

Quiz 6.1–6.2 (p. 427)
1. \(36^{2/3} = (\sqrt[3]{36})^2 = 6^2 = 216\)
2. \(64^{-2/3} = \frac{1}{64^{2/3}} = \frac{1}{(\sqrt[3]{64})^2} = \frac{1}{4^2} = \frac{1}{16}\)
3. \(-625^{1/4} = -[(\sqrt[4]{625})^{1/4}] = -5^2 = -125\)
4. \((-32)^{2/3} = (\sqrt[3]{-32})^2 = (-2)^2 = 4\)
5. \(x^4 = 20\)
6. \(x^5 = -10\)
   \[x = \pm \sqrt[5]{20}\]
   \[x = -\sqrt[5]{10}\]
7. \(x^6 = 26\)
8. \((x + 3)^3 = -16\)
   \[x^3 = 21\]
   \[x = \pm \sqrt[3]{21}\]
   \[x = \sqrt[3]{16} - 3\]
9. \(\sqrt[3]{32} \cdot \sqrt[3]{8} = \sqrt[3]{32 \cdot 8} = \sqrt[3]{256} = 4\)
10. \((\sqrt[10]{10} \cdot \sqrt[10]{10})^6 = (10^{1/10} \cdot 10^{1/10})^6 = 10^{(1/10 + 1/10)(6)} = 10^{11/10} = 10^{1.1}\)

Lesson 6.3

6.3 Guided Practice (pp. 429–431)
1. \(f(x) + g(x) = -2x^{2/3} + 7x^{2/3} = (-2 + 7)x^{2/3} = 5x^{2/3}\)
2. \(f(x) - g(x) = -2x^{2/3} - 7x^{2/3} = (-2 - 7)x^{2/3} = -9x^{2/3}\)
3. The functions \(f\) and \(g\) each have the same domain: all real numbers. So, the domains of \(f + g\) and \(f - g\) also consist of all real numbers.
4. \(f(x) \cdot g(x) = 3x(x^{1/3}) = 3x^{(1 + 1/3)} = 3x^{4/3}\)
5. \(\frac{f(x)}{g(x)} = \frac{3x}{x^{1/3}} = 3x^{(1 - 1/3)} = 3x^{2/3}\)
6. The functions \(f\) and \(g\) each have the same domain: all real numbers. So, the domain of \(f \cdot g\) also consists of all real numbers. Because \(g(0) = 0\), the domain of \(f \cdot g\) is restricted to all real numbers except \(x = 0\).
7. \(r(m) \cdot s(m) = \left(1.446 \times 10^3\right) m^{-0.05} = \left(1.446 \times 10^3\right) \left(1.7 \times 10^{-5}\right)^{-0.05} = \left(1.446 \times 10^3\right)(0.55) = 791,855.335\)
   The white rhino has about 791,855.335 heartbeats over its lifetime.
8. \(g(f(5))\)
   \(f(5) = 3(5) - 8 = 7\)
   \(g(7) = 2^2 = 4\)
9. \(f(5) = 5(5) = 25\)
   \(g(25) = 5^2 = 25\)
10. \(f(5) = 3(5) - 8 = 7\)
    \(g(7) = 2^2 = 4\)
    \(g(4) = 2^2 = 4\)
11. \(g(f(5))\)
    \(f(5) = 3(5) - 8 = 7\)
    \(g(7) = 2^2 = 4\)
    \(g(4) = 2^2 = 4\)
12. \(f(2x^{-1})\)
    \(g(x) = 2x + 7\)
    \(f(2x^{-1}) = 2(2x^{-1}) + 7 = \frac{2}{x} + 7\)
    \(g(2x^{-1}) = 2(2x^{-1}) - 7 = 4\)
13. \(f(2x^{-1}) = 2(2x^{-1}) + 7 = 4\)
   \(f(x) = f(2x^{-1}) = 2(2x^{-1}) + 7 = 4x^{-1} + 7 = \frac{4}{x} + 7\)
   \(g(x) = 2x^{-1} = 2(2x^{-1}) - 7 = 2(2x^{-1}) - 7 = 2x - 7\)
   \(f(x) = f(2x^{-1}) = 2(2x^{-1}) + 7 = 2x + 7\)
   \(f(x) = f(2x^{-1}) = 2(2x^{-1}) + 7 = 2x - 7\)
   \(g(x) = 2x^{-1} = 2(2x^{-1}) + 7 = 2x + 7\)
   \(f(x) = f(2x^{-1}) = 2(2x^{-1}) - 7 = 2x - 7\)
   \(g(x) = 2x^{-1} = 2(2x^{-1}) + 7 = 2x + 7\)
14. \(f(x) = f(2x^{-1}) = 2(2x^{-1}) - 7 = 2x - 7\)
   \(g(x) = 2x^{-1} = 2(2x^{-1}) - 7 = 2x - 7\)
   \(f(x) = f(2x^{-1}) = 2(2x^{-1}) + 7 = 2x + 7\)
   \(g(x) = 2x^{-1} = 2(2x^{-1}) - 7 = 2x - 7\)
   \(f(x) = f(2x^{-1}) = 2(2x^{-1}) + 7 = 2x + 7\)
   \(g(x) = 2x^{-1} = 2(2x^{-1}) - 7 = 2x - 7\)
   \(f(x) = f(2x^{-1}) = 2(2x^{-1}) - 7 = 2x - 7\)
   \(g(x) = 2x^{-1} = 2(2x^{-1}) + 7 = 2x + 7\)
15. Three labels A, B, C, are added to the graph to indicate the three right triangles.

   For right triangle A
   \[a^2 + b^2 = c^2\]
   \[(8 - 2)^2 + 8^2 = c^2\]
   \[6^2 + 8^2 = c^2\]
   \[100 = c^2\]
   \[10 = c\]

   For right triangle B
   \[a^2 + b^2 = c^2\]
   \[a^2 + b^2 = c^2\]
   \[4^2 + 8^2 = c^2\]
   \[80 = c^2\]
   \[4\sqrt{5} = c\]
   \[2\sqrt{5} = c\]

   The perimeter of the right triangle is
   \[10 + 4\sqrt{5} + 2\sqrt{5} = 10 + 6\sqrt{5}\]
6.3 Exercises (pp. 432–434)

Skill Practice

1. The function \( h(x) = g(f(x)) \) is called the composition of the function \( g \) with the function \( f \).
   
   Sample answer:
   
   \[
   f(x) = 2x^{1/3}, \quad g(x) = 4x^{-1/3}
   \]
   
   \[
   f(x) + g(x) = 2x^{1/3} + 4x^{-1/3}
   \]
   
   Domain: all nonnegative real numbers

2. The sum of two power functions is sometimes a power function.

3. \( f(x) + g(x) = -(3x^{1/3} + 4x^{1/2} + 5x^{1/3} + 4x^{1/2}) = -3 + 5x^{1/3} + (4 + 4)x^{1/2} = 2x^{1/3} + 8x^{1/2} \)
   
   Domain: all nonnegative real numbers

4. \( g(x) + f(x) = 5x^{1/3} + 4x^{1/2} - 3x^{1/3} + 4x^{1/2} \)
   
   Domain: all nonnegative real numbers

5. \( f(x) + g(x) = -3x^{1/3} + 4x^{1/2} - (-3x^{1/3} + 4x^{1/2}) = -3 + 5x^{1/3} + (4 + 4)x^{1/2} \)
   
   Domain: all nonnegative real numbers

6. \( g(x) + f(x) = -3x^{1/3} + 4x^{1/2} - (5x^{1/3} + 4x^{1/2}) = -3 + 5x^{1/3} + (4 + 4)x^{1/2} \)
   
   Domain: all nonnegative real numbers

7. \( f(x) - g(x) = -3x^{1/3} + 4x^{1/2} - (5x^{1/3} + 4x^{1/2}) \)
   
   Domain: all nonnegative real numbers

8. \( g(x) - f(x) = 5x^{1/3} + 4x^{1/2} - (5x^{1/3} + 4x^{1/2}) \)
   
   Domain: all nonnegative real numbers

9. \( f(x) - f(x) = -3x^{1/3} + 4x^{1/2} - (-3x^{1/3} + 4x^{1/2}) = 0 \)
   
   Domain: all nonnegative real numbers

10. \( g(x) - g(x) = 5x^{1/3} + 4x^{1/2} - (5x^{1/3} + 4x^{1/2}) \)
    
    Domain: all nonnegative real numbers

11. \( B: f(x) + g(x) = -7x^{2/3} - 1 + 2x^{2/3} + 6 \)
    
    Domain: all nonnegative real numbers

12. \( f(x) \cdot g(x) = 4x^{2/3} \cdot 5x^{1/2} \)
    
    Domain: all real numbers

13. \( g(x) \cdot f(x) = 5x^{1/2} \cdot 4x^{2/3} \)
    
    Domain: all nonnegative real numbers

14. \( f(x) \cdot f(x) = 4x^{2/3} \cdot 4x^{2/3} \)
    
    Domain: all positive real numbers

15. \( g(x) \cdot g(x) = 5x^{1/2} \cdot 5x^{1/2} \)
    
    Domain: all nonnegative real numbers

16. \( \frac{f(x)}{g(x)} = \frac{4x^{2/3}}{5x^{1/2}} = \frac{4x^{2/3 - 1/2}}{5} = \frac{4x^{1/6}}{5} \)
    
    Domain: all positive real numbers

17. \( \frac{g(x)}{f(x)} = \frac{5x^{1/2}}{4x^{2/3}} = \frac{5x^{1/2 - 2/3}}{4} = \frac{5x^{-1/6}}{4} = \frac{5}{4x^{1/6}} \)
    
    Domain: all positive real numbers

18. \( \frac{f(x)}{f(x)} = \frac{4x^{2/3}}{4x^{2/3}} = 1 \)
    
    Domain: all positive real numbers

19. \( \frac{g(x)}{g(x)} = \frac{5x^{1/2}}{5x^{1/2}} = 1 \)
    
    Domain: all positive real numbers

20. \( g(-3) = -(3)^2 = -9 \)

21. \( f(2) = 3(2) + 2 = 8 \)

\[
\frac{g(f(2))}{g(8)} = \frac{-8^2}{-64} = -4
\]
Chapter 6, continued

22. \( f(-9) = 3(-9) + 2 = -25 \)
\[ h(f(-9)) = h(-25) = \frac{-25 - 2}{5} = \frac{-27}{5} \]

23. \( h(8) = \frac{8 - 2}{5} = \frac{6}{5} \)
\[ g(h(8)) = g\left(\frac{6}{5}\right) = \frac{-6^2}{25} = \frac{-36}{25} \]

24. \( g(5) = -5^2 = -25 \)
\[ h(g(5)) = h(-25) = \frac{-25 - 2}{5} = \frac{-27}{5} \]

25. \( f(7) = 3(7) + 2 = 23 \)
\[ f(f(7)) = f(23) = 3(23) + 2 = 71 \]

26. \( h(-4) = \frac{-4 - 2}{5} = \frac{-6}{5} \)
\[ h(h(-4)) = h\left(\frac{-6}{5}\right) = \frac{-6 - 2}{5} = \frac{-16}{25} \]

27. \( g(-5) = (-5)^2 = 25 \)
\[ g(g(-5)) = g(-25) = -(25)^2 = -625 \]

28. \[ f(2x - 7) = 3(2x - 7)^{-1} = \frac{3}{2x - 7} \]

The domain of \( f(2x) \) consists of all real numbers except \( x = \frac{7}{2} \) because \( 2x = 7 \) is not in the domain of \( f \).

29. \( g(3x^{-1}) = 2(3x^{-1}) - 7 = 6x^{-1} - 7 = \frac{6}{x} - 7 \)

The domain of \( g(3x^{-1}) \) consists of all real numbers except \( x = 0 \) because 0 is not in the domain of \( f \).

30. \[ h(3x^{-1}) = \frac{3x^{-1} + 4}{3} = \frac{3x^{-1} + 4}{3} = \frac{1}{x} + \frac{4}{3} \]

The domain of \( h(3x^{-1}) \) consists of all real numbers except \( x = 0 \) because 0 is not in the domain of \( f \).

31. \[ g\left(\frac{x + 4}{3}\right) = \frac{3x + 4}{3} - 7 = \frac{2x + 8 - 21}{3} = \frac{2x - 13}{3} \]

The domain of \( g\left(\frac{x + 4}{3}\right) \) consists of all real numbers.

32. \[ h(2x - 7) = \frac{2x - 7 + 4}{3} = \frac{2x - 3}{3} \]

The domain of \( h(2x - 7) \) consists of all real numbers.

33. \[ f(3x^{-1}) = 3(3x^{-1})^{-1} = 3(3^{-1}) = 3x = x \]

The domain of \( f(3x^{-1}) \) consists of all real numbers except \( x = 0 \), because 0 is not in the domain of \( f \).

34. \[ h\left(\frac{x + 4}{3}\right) = \frac{x + 4}{3} + \frac{4}{3} = \frac{x + 12}{3} = \frac{x + 16}{9} \]

The domain of \( h\left(\frac{x + 4}{3}\right) \) consists of all real numbers.

35. \[ g(2x - 7) = 2(2x - 7) - 7 = 4x - 14 - 7 = 4x - 21 \]

The domain of \( g(2x - 7) \) consists of all real numbers.

36. When performing \( f(4x) \), \( 4x \) should have been substituted for \( x \) in the function \( f \).
\[ f(g(x)) = f(4x) = (4x)^2 - 3 = 16x^2 - 3 \]

37. The product \( 4(x^2 - 3) \) was not performed correctly.
\[ g(f(x)) = g(x^2 - 3) = 4(x^2 - 3) = 4x^2 - 12 \]

38. \[ A; \]
\[ g(f(x)) = g(7x^2) = 3(7x^2)^{-2} = 3(7^{-2}x^{-4}) = \frac{3}{49x^4} \]

39. Sample answer: \( f(x) = x \), \( g(x) = x^{-1} \)
\[ f(g(x)) = f(x) \]
\[ f(x^{-1}) = g(x) \]
\[ x^{-1} = x^{-1} \]

40. Sample answer: \( f(x) = \sqrt[5]{x} \), \( g(x) = x + 2 \)
\[ h(x) = f(g(x)) = f(x + 2) = \sqrt[5]{x + 2} \]

41. Sample answer: \( f(x) = \frac{4}{x + 7} \), \( g(x) = 3x^2 \)
\[ h(x) = f(g(x)) = f(3x^2) = \frac{4}{3x^2 + 7} \]

42. Sample answer: \( f(x) = |x| \), \( g(x) = 2x + 9 \)
\[ h(x) = f(g(x)) = f(2x + 9) = |2x + 9| \]

Problem Solving

43. \[ r(w) = \frac{(1.1w)^{0.734}}{b(w) - d(w)} \]
\[ = \frac{1.1w^{0.734}}{0.007w - 0.002w} \]
\[ = \frac{1.1w^{0.734}}{0.005w} \]
\[ = \frac{220w^{0.734}}{0.005w} \]
\[ = 220w^{0.734} \]
\[ r(w) = 220w^{0.734} - 0.266 \approx 134 \]

The breathing rate of a mammal that weighs 6.5 grams is about 134 breaths per minute.
\[ r(300) = 220(300)^{0.266} \approx 48.3 \]

The breathing rate of a mammal that weighs 300 grams is about 48.3 breaths per minute.
\[ r(70,000) = 220(70,000)^{0.266} \approx 11.3 \]

The breathing rate of a mammal that weighs 70,000 grams is about 11.3 breaths per minute.

44. \[ C(x(t)) = C(50t) + 750 = 3000 + 750 = 3750 \]
\[ C(x(5)) = 3000(5) + 750 = 15,750 \]

This number represents the cost ($15,750) of 5 hours of production in the factory.

45. Let \( x \) represent the regular price.

Function for $15 discount: \( f(x) = x - 15 \)

Function for 10% discount: \( g(x) = x - 0.1x = 0.9x \)

a. \[ g(f(x)) = g(x - 15) = 0.9(x - 15) \]
\[ g(85) = 0.9(85 - 15) = 0.9(70) = 63 \]

The sale price is $63 when the $15 discount is applied before the 10% discount.

b. \[ f(g(x)) = f(0.9x) = 0.9x - 15 \]
\[ f(85) = 0.9(85) - 15 = 76.5 - 15 = 61.50 \]

The sale price is $61.50 when the 10% discount is applied before the $15 discount.
Chapter 6, continued

e. If the 10% discount is applied before the $15 discount, you get a better deal. Your purchase will be $61.50 instead of $63.

46. a. Distance from point $A$ to point $D$: $20 - x$

Distance = rate $\times$ time

$20 - x = (6.4)r(x)$

Distance from point $D$ to point $B$:

$x^2 + 12^2 = c^2$

$x^2 + 144 = c^2$

$\sqrt{x^2 + 144} = c$

Distance = rate $\times$ time

$\sqrt{x^2 + 144} = (0.9)s(x)$

$\sqrt{x^2 + 144} = \frac{0.9}{s(x)}$

b. $t(x) = r(x) + s(x) = \frac{20 - x}{6.4} + \sqrt{x^2 + 144} = \frac{0.9}{s(x)}$

c. The value of $x$ that minimizes $t(x)$ is 1.7. This means that to get to the ball in the shortest time, Elvis should run along the beach $20 - 1.7 = 18.3$ meters and then swim out to the ball.

47. a. $f(1) = \frac{1 + 2}{2} = \frac{3}{2} = 1.5$

$f(f(1)) = f(1.5) = \frac{1.5 + 2}{2} = 1.417$

$f(f(f(1))) = f(1.417) = \frac{1.417 + 2}{2} = 1.417$

$f(f(f(f(1)))) = f(1.414) = \frac{1.414 + 2}{2} = 1.414214$

b. $f(f(f(f(f(1))))) = f(1.414214) = \frac{1.414214 + 2}{2} = 1.414214$

$\sqrt{2} = 1.414213562$

You need to compose the function 3 times in order for the result to approximate $\sqrt{2}$ to three decimal places. You need to compose the function 4 times in order for the result to approximate $\sqrt{2}$ to six decimal places.

Mixed Review

48. $y - 2x = 12$

$y = 2x + 12$

49. $3x - 2y = 10$

$-2y = -3x + 10$

$y = \frac{3}{2}x - 5$

50. $x = -3y + 9$

$3y = -x + 9$

$y = -\frac{1}{3}x + 3$

51. $3x - 4y = 7$

$-4y = -3x + 7$

$y = \frac{3}{4}x - \frac{7}{4}$

52. $x - y = 12$

$-y = -x + 12$

$y = x - 12$

54. $\begin{array}{c}
\text{a.} \\
\text{b.} \\
\text{c.}
\end{array}$

55. $\begin{array}{c}
\text{a.} \\
\text{b.} \\
\text{c.}
\end{array}$

56. $\begin{array}{c}
\text{a.} \\
\text{b.} \\
\text{c.}
\end{array}$

57. $\begin{array}{c}
\text{a.} \\
\text{b.} \\
\text{c.}
\end{array}$

58. $\begin{array}{c}
\text{a.} \\
\text{b.} \\
\text{c.}
\end{array}$

59. $\begin{array}{c}
\text{a.} \\
\text{b.} \\
\text{c.}
\end{array}$

60. $\begin{array}{c}
\text{a.} \\
\text{b.} \\
\text{c.}
\end{array}$

61. $\begin{array}{c}
\text{a.} \\
\text{b.} \\
\text{c.}
\end{array}$

62. $\begin{array}{c}
\text{a.} \\
\text{b.} \\
\text{c.}
\end{array}$

63. $\begin{array}{c}
\text{a.} \\
\text{b.} \\
\text{c.}
\end{array}$

64. $\begin{array}{c}
\text{a.} \\
\text{b.} \\
\text{c.}
\end{array}$

65. $\begin{array}{c}
\text{a.} \\
\text{b.} \\
\text{c.}
\end{array}$
Chapter 6, continued

66.  

Graphing Calculator Activity 6.3 (p. 435)

1.  

2.  

3.  

4.  

Mixed Review of Problem Solving (p. 436)

1. a. \( s(x) = x^2 \)
   
   b. \( c(x) = \pi \left( \frac{x}{2} \right)^2 = \frac{\pi x^2}{4} \)
   
   c. \( r(x) = x^2 - \pi \left( \frac{x}{2} \right)^2 = 1 - \frac{\pi}{4} x^2 = 0.21x^2 \)

2. a. \( V = 3^{-1} (4\pi)^{1/2}(S^3)^{1/2} \)
   
   \[ = \frac{S^{3/2}}{3(2\pi^{1/2})} \]
   
   \[ = \frac{S\sqrt{S}}{6\pi^{1/2}} \]
   
   \[ = \frac{\sqrt{S}}{6\pi} \]
   
   b. \( V = 79\sqrt{\pi(79)} = 66 \)

The volume of the candlepin bowling ball is about 66 cubic inches.

3. \( f(x) = x - 100,000, \ g(x) = 0.03x \)
   
   \[ f(g(x)) = 0.03x - 100,000 \]
   
   \[ g(f(x)) = 0.03(x - 100,000) \]

The composition \( f(g(x)) \) represents your bonus if \( x > 100,000 \), because the bonus must be applied after $100,000 is subtracted.

4. a. \( V = \pi r^2 h = 3.14x^2(5) = 15.7x^2 \)
   
   b. \( \frac{15.7x^2}{2} = 128(8.8) \)
   
   \[ = 15.7x^2 = 2252.8 \]
   
   c. \( 15.7x^2 = 2252.8 \)
   
   \[ x^2 = \frac{2252.8}{15.7} = 143.49 \]
   
   \[ x = \pm \sqrt{143.49} \]
   
   \[ x = \pm 12 \]

The radius of the pool is about 12 feet.

b. \( \frac{4}{5} - \frac{1}{2} = \frac{3}{10} \)

Volume filled = Hose output \cdot Time

\[ \frac{3}{10} (2252.8) = (104 + 128) \cdot t \]

\[ 675.84 = 232t \]

\[ 2.9 = t \]

It will take a total of 8.8 + 2.9, or 11.7 hours to fill \( \frac{4}{5} \) of the pool.

5. Sample answer:
   
   \[ f(x) = 5 \]
   
   \[ f(x) = x \]
   
   \[ f(f(x)) = f(5) = 5 \]
   
   \[ f(f(x)) = f(x) = x \]

6. Sample answer:
   
   \[ \left[ 16^{1/2} \right] = \left( \frac{4}{2} \right)^{1/2} \]
   
   \[ = 2^{5/2} \]
   
   Divide 4 by 2.
   
   \[ = 32 \]
   
   Evaluate \( 2^{5/2} \).

Yes, there is another set of steps you could use to simplify the expression. For example:

\[ \left[ 16^{1/2} \right] = \left( \frac{16}{4} \right)^{1/2} \]

Power of a quotient property

\[ = (4^{1/2})^{1/2} \]

Divide 16 by 4.

\[ = 4^{1/2} \cdot 5 \]

Power of a power property

\[ = 4^{5/2} \]

Multiply \( \frac{1}{2} \) by 5.

\[ = 32 \]

Evaluate \( 4^{5/2} \).
Chapter 6, continued

7. \( V = \frac{4}{3} \pi r^3 \)

900 = \( \frac{4}{3}(3.14)r^3 \)

900 = 4.19\( r^3 \)

214.8 = \( r^3 \)

6.0 = \( r \)

The radius of the sphere is about 6.0 inches.

Lesson 6.4

Investigating Algebra Activity 6.4 (p. 437)

1. a. \( f(x) = 3x + 2 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = f(x) )</td>
<td>-4</td>
<td>-1</td>
<td>2</td>
<td>5</td>
<td>8</td>
</tr>
</tbody>
</table>

\((2, 0), (5, 1)\)

\( m = \frac{1 - 0}{5 - 2} = \frac{1}{3} \)

\( y - y_1 = m(x - x_1) \)

\( y - 0 = \frac{1}{3}(x - 2) \)

\( y = \frac{x - 2}{3} \)

\( g(x) = \frac{x - 2}{3} \)

1. b. \( f(x) = \frac{x - 1}{6} \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>-5</th>
<th>-2</th>
<th>1</th>
<th>4</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = f(x) )</td>
<td>-1</td>
<td>1/2</td>
<td>0</td>
<td>1/2</td>
<td>1</td>
</tr>
</tbody>
</table>

\((0, 1), (1, 7)\)

\( m = \frac{7 - 1}{1 - 0} = 6 \)

\( y - y_1 = m(x - x_1) \)

\( y - 1 = 6(x - 0) \)

\( y - 1 = 6x \)

\( y = 6x + 1 \)

\( g(x) = 6x + 1 \)

2. a. You can graph the inverse of a function by reflecting it in the line \( y = x \).

3. a. In words, \( g \) is the function that subtracts 2 from \( x \) then divides the result by 3.

\( f(g(x)) = f\left(\frac{x - 2}{3}\right) \quad g(f(x)) = g(3x + 2) \)

\( = \frac{3\left(\frac{x - 2}{3}\right) + 2}{3x + 2 - 2} \)

\( = \frac{x - 2 + 2}{3} \)

\( = \frac{x}{3} \)

\( = x \)

If \( f(g(x)) = x \) and \( g(f(x)) = x \), then the function is indeed the inverse of the original function.

2. b. You can graph the inverse of a function by reflecting it in the line \( y = x \).

3. b. In words, \( g \) is the function that multiplies \( x \) by 6 and then adds 1.

\( f(g(x)) = f(6x + 1) \quad g(f(x)) = g\left(\frac{x - 1}{6}\right) \)

\( = 6x + 1 - 1 \)

\( = 6x \)

\( = x \)

If \( f(g(x)) = x \) and \( g(f(x)) = x \), then the function is indeed the inverse of the original function.

1. c. \( f(x) = 4 - \frac{3}{2}x \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = f(x) )</td>
<td>-1</td>
<td>1</td>
<td>3</td>
<td>-1</td>
<td>-3</td>
</tr>
</tbody>
</table>

\((4, 0), (1, 2)\)

\( m = \frac{2 - 0}{1 - 0} = \frac{2}{3} \)

\( y - y_1 = m(x - x_1) \)

\( y - 0 = \frac{2}{3}(x - 4) \)

\( y = \frac{-2(x - 4)}{3} \)

\( = \frac{-2x + 8}{3} \)

\( g(x) = \frac{-2x + 8}{3} \)

2. c. You can graph the inverse of a function by reflecting it in the line \( y = x \).

3. c. In words, \( g \) is the function that multiplies \( x \) by \(-2\) and then adds 8, dividing the result by 3.

\( f(g(x)) = f\left(\frac{-2x + 8}{3}\right) \quad g(f(x)) = g\left(\frac{4 - \frac{3x}{2}}{3}\right) \)

\( = 4 - \frac{3}{2}\left(\frac{-2x + 8}{3}\right) \)

\( = \frac{-2(4 - \frac{3x}{2}) + 8}{3} \)

\( = 4 - \frac{8 + 3x + 8}{3} \)

\( = 4 - (-x + 4) \)

\( = \frac{3x}{3} \)

\( = 4 + x - 4 \)

\( = x \)

\( = x \)

If \( f(g(x)) = x \) and \( g(f(x)) = x \), then the function is indeed the inverse of the original function.

6.4 Guided Practice (pp. 439–442)

1. \( f(x) = x + 4 \)

\( y = x + 4 \)

\( x = y + 4 \)

\( x - 4 = y \)

\( f^{-1}(x) = x - 4 \)

\( f(f^{-1}(x)) = f(x - 4) \quad f^{-1}(f(x)) = f^{-1}(x + 4) \)

\( = x - 4 + 4 \)

\( = x + 4 - 4 \)

\( = x \checkmark \)

\( = x \checkmark \)
Chapter 6, continued

2. \( f(x) = 2x - 1 \)
   \( y = 2x - 1 \)
   \( x = 2y - 1 \)
   \( x + 1 = 2y \)
   \( \frac{1}{2}x + \frac{1}{2} = y \)
   \( f^{-1}(x) = \frac{1}{2}x + \frac{1}{2} \)
   \( f(f^{-1}(x)) = f(\frac{1}{2}x + \frac{1}{2}) \)
   \( f^{-1}(f(x)) = f^{-1}(2x - 1) \)
   \( = \frac{1}{2}(2x - 1) + \frac{1}{2} \)
   \( = x + 1 - 1 \)
   \( = x \checkmark \)

3. \( f(x) = -3x + 1 \)
   \( y = -3x + 1 \)
   \( x = 3y + 1 \)
   \( x - 1 = -3y \)
   \( \frac{1}{3}x + \frac{1}{3} = y \)
   \( f^{-1}(x) = \frac{1}{3}x + \frac{1}{3} \)
   \( f(f^{-1}(x)) = f(-3x + 1) \)
   \( = -3\left(-3x + 1\right) + \frac{1}{3} \)
   \( = x - 1 + 1 \)
   \( = x \checkmark \)

4. \( L = \frac{8}{3}R^{13} + \frac{40}{3} + \frac{8}{3}(13) + \frac{104}{3} + \frac{40}{3} = 144 \)  
   The band provides 13 pounds of resistance when it is stretched to 48 inches.

5. \( f(x) = x^6 \), \( x \geq 0 \)
   \( f(x) = x^6 \)
   \( y = x^6 \)
   \( x = y^6 \)
   \( \sqrt[6]{x} = y \)
   \( f^{-1}(x) = \sqrt[6]{x} \)

6. \( g(x) = \frac{1}{27}x^3 \)
   \( y = \frac{1}{27}x^3 \)
   \( x = \frac{1}{27}y^3 \)
   \( 27x = y^3 \)
   \( \sqrt[3]{27x} = y \)
   \( 3\sqrt[3]{x} = y \)
   \( g^{-1}(x) = 3\sqrt[3]{x} \)

7. \( f(x) = \frac{64}{125}x^3 \)
   \( y = \frac{64}{125}x^3 \)
   \( x = \frac{64}{125}y^3 \)
   \( \frac{125}{64}x = y^3 \)
   \( \sqrt[3]{\frac{125}{64}x} = y \)
   \( \frac{5\sqrt[3]{x}}{4} = y \)
   \( f^{-1}(x) = \frac{5\sqrt[3]{x}}{4} \)

8. \( f(x) = -x^3 + 4 \)
   \( y = -x^3 + 4 \)
   \( x = -y^3 + 4 \)
   \( x - 4 = -y^3 \)
   \( 4 - x = y^3 \)
   \( \sqrt[3]{4 - x} = y \)
   \( f^{-1}(x) = \sqrt[3]{4 - x} \)

9. \( f(x) = 2x^2 + 3 \)
   \( y = 2x^2 + 3 \)
   \( x = 2y^2 + 3 \)
   \( x - 3 = 2y \)
   \( \frac{1}{2}x - \frac{3}{2} = y^5 \)
   \( \sqrt[5]{\frac{1}{2}x - \frac{3}{2}} = y \)
   \( f^{-1}(x) = \sqrt[5]{\frac{1}{2}x - \frac{3}{2}} \)

10. \( g(x) = -7x^5 + 7 \)
    \( y = -7x^5 + 7 \)
    \( x = -7y^5 + 7 \)
    \( x - 7 = -7y^5 \)
    \( 1 - \frac{x}{7} = y^5 \)
    \( \sqrt[5]{1 - \frac{x}{7}} = y \)
    \( g^{-1}(x) = \sqrt[5]{1 - \frac{x}{7}} \)

11. \( P = 10.7t^{0.272} \)
    \( \frac{P}{10.7} = t^{0.272} \)
    \( \left(\frac{P}{10.7}\right)^{1.0272} = (t^{0.272})^{1.0272} \)
    \( \left(\frac{P}{10.7}\right)^{1.68} = t \)

When \( P = 25 \):
\[ t = \frac{25^{1.68}}{10.7} = 22.7 \]

You can predict that the average ticket price will reach $25 about 22 years after 1995, or in 2017.
Chapter 6, continued

6.4 Exercises (pp. 442–445)

Skill Practice

1. An inverse relation interchanges the input and output values of the original relation.

2. A function $g$ is an inverse of $f$ provided $f(g(x)) = x$ and $g(f(x)) = x$.

3. $y = 4x - 1$
   $x = 4y - 1$
   $x + 1 = 4y$
   $\frac{1}{4}x + \frac{1}{4} = y$

4. $y = -2x + 5$
   $x = -2y + 5$
   $x - 5 = -2y$
   $\frac{1}{2}x + \frac{5}{2} = y$

5. $y = 7x - 6$
   $x = 7y - 6$
   $x + 6 = 7y$
   $\frac{1}{2}x + \frac{6}{7} = y$

6. $y = 10x - 28$
   $x = 10y - 28$
   $x + 28 = 10y$
   $\frac{1}{10}x + \frac{14}{5} = y$

7. $y = 12x + 7$
   $x = 12y + 7$
   $x - 7 = 12y$
   $\frac{1}{12}x - \frac{7}{12} = y$

8. $y = -18x - 5$
   $x = -18y - 5$
   $x + 5 = -18y$
   $\frac{1}{18}x - \frac{5}{18} = y$

9. $y = 5x + \frac{1}{3}$
   $x = 5y + \frac{1}{3}$
   $x - \frac{1}{5} = 5y$
   $\frac{1}{5}x - \frac{1}{15} = y$

10. $y = \frac{-3x + 7}{5}$
    $x = \frac{-3y + 7}{5}$
    $x - \frac{7}{5} = \frac{-3y}{5}$
    $\frac{5}{3}x + \frac{7}{3} = y$

11. $y = 6x - 11$
    $x = 6y - 11$
    $x + 11 = 6y$
    $\frac{5}{6} + \frac{11}{6} = x$

12. In the last step, each term was not divided by 6.

13. In the first step, the terms involving $x$ and $y$ were switched instead of just the variables $x$ and $y$.

14. Sample answer:

   $f^{-1}(x) = 3x - 1$
   $y = 3x - 1$
   $x = 3y - 1$
   $x + 1 = 3y$
   $\frac{1}{3}x + \frac{1}{3} = y$
   $f(x) = \frac{1}{3}x + \frac{1}{3}$

15. $f(x) = x + 4$, $g(x) = x - 4$

   $f(g(x)) = f(x - 4)$
   $g(f(x)) = g(x + 4)$
   $= x - 4 + 4$
   $= x + 4 - 4$
   $= x$

16. $f(x) = 2x + 3$, $g(x) = \frac{1}{2}x - \frac{3}{2}$

   $f(g(x)) = f\left(\frac{1}{2}x - \frac{3}{2}\right)$
   $g(f(x)) = g(2x + 3)$
   $= 2\left(\frac{1}{2}x - \frac{3}{2}\right) + 3$
   $= x - 3 + 3$
   $= x + \frac{3}{2} - \frac{3}{2}$
   $= x$

17. $f(x) = \frac{1}{4}x^3$, $g(x) = (4x)^{1/3}$

   $f(g(x)) = f[(4x)^{1/3}]$
   $g(f(x)) = g(\left(\frac{1}{4}x\right)^{1/3})$
   $= \frac{1}{4}(4x)^{1/3}$
   $= \frac{1}{4}(4x)$
   $= (x)^{1/3}$
   $= x$

18. $f(x) = \frac{1}{5}x - 1$, $g(x) = 5x + 5$

   $f(g(x)) = f(5x + 5)$
   $g(f(x)) = g\left(\frac{1}{5}x - 1\right)$
   $= \frac{1}{5}(5x + 5) - 1$
   $= \frac{5}{3}x - 1 + 5$
   $= x + 1 - 1$
   $= x - 5 + 5$
   $= x$

19. $f(x) = 4x + 9$, $g(x) = \frac{1}{4}x^2 - \frac{9}{4}$

   $f(g(x)) = f\left(\frac{1}{4}x^2 - \frac{9}{4}\right)$
   $g(f(x)) = g(4x + 9)$
   $= 4\left(\frac{1}{4}x^2 - \frac{9}{4}\right) + 9$
   $= \frac{1}{4}(4x + 9) - \frac{9}{4}$
   $= x - 9 + 9$
   $= x + 9 - \frac{9}{4}$
   $= x$

20. $f(x) = 5x^2 - 2$, $x \geq 0$; $g(x) = \left(\frac{x + 2}{3}\right)^{1/2}$

   $f(g(x)) = f\left(\left(\frac{x + 2}{3}\right)^{1/2}\right)$
   $g(f(x)) = g(5x^2 - 2)$
   $= 5\left(\frac{x + 2}{3}\right)^{1/2} - 2$
   $= \left(\frac{5x^2}{3}ight)^{1/2}$
   $= \frac{5}{3}x - 2$
   $= (x)^{1/2}$
   $= x$

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Chapter 6, continued

21. B;
\[ y = mx + b \]
\[ m = \frac{2}{3}, b = -4 \]
\[ y = \frac{2}{3}x - 4 \]
\[ x + 4 = \frac{2}{3}y \]
\[ \frac{3}{2}x + 6 = g(x) \]

22. \( f(x) = x^7 \)
\[ y = x^7 \]
\[ x = y^7 \]
\[ \sqrt[7]{x} = y \]
\[ f^{-1}(x) = \sqrt[7]{x} \]

23. \( f(x) = 4x^4, x \geq 0 \)
\[ y = 4x^4 \]
\[ x = 4y^4 \]
\[ \sqrt[4]{\frac{x}{4}} = y \]
\[ \pm \sqrt[4]{\frac{x}{4}} = y \]
\[ f^{-1}(x) = \frac{\sqrt[4]{4x}}{2} \]

24. \( f(x) = -10x^6, x \leq 0 \)
\[ y = -10x^6 \]
\[ x = -\frac{10y^6}{10} \]
\[ \pm \sqrt[6]{\frac{-10y^6}{10}} = y \]
\[ \pm \sqrt[6]{-100,000y} = y \]
\[ f^{-1}(x) = -\frac{\sqrt[6]{-100,000x}}{10} \]

25. \( f(x) = 32x^3 \)
\[ y = 32x^3 \]
\[ x = \frac{y}{32} \]
\[ \sqrt[3]{\frac{x}{32}} = y \]
\[ \sqrt[3]{\frac{y}{32}} = y \]
\[ \sqrt[3]{\frac{y}{32}} = y \]
\[ f^{-1}(x) = \frac{\sqrt[3]{x}}{2} \]

26. \( f(x) = \frac{2}{3}x^3 \)
\[ y = \frac{2}{3}x^3 \]
\[ x = \frac{2}{3}y^3 \]
\[ \frac{3}{2}x = y^3 \]
\[ \frac{\sqrt{3x}}{2} = y \]

27. \( f(x) = \frac{16}{25}x^2, x \leq 0 \)
\[ y = \frac{2}{5}x^3 \]
\[ x = \frac{2}{5}y^3 \]
\[ \frac{5}{2}x = y^3 \]
\[ \frac{\sqrt{5x}}{2} = y \]

28. C;

29. Because no horizontal line intersects the graph of \( f \) more than once, the inverse of \( f \) is a function.

30. Because no horizontal line intersects the graph of \( f \) more than once, the inverse of \( f \) is a function.

31. Because a horizontal line intersects the graph of \( f \) more than once, the inverse of \( f \) is not a function.
Because no horizontal line intersects the graph of \( f \) more than once, the inverse of \( f \) is a function.

Because no horizontal line intersects the graph of \( f \) more than once, the inverse of \( f \) is a function.

Because no horizontal line intersects the graph of \( f \) more than once, the inverse of \( f \) is a function.

Because a horizontal line intersects the graph of \( f \) more than once, the inverse of \( f \) is not a function.

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Because a horizontal line intersects the graph of \( f \) more than once, the inverse of \( f \) is not a function.

Because a horizontal line intersects the graph of \( f \) more than once, the inverse of \( f \) is not a function.

\[
f(x) = \frac{3}{2}x^4, \quad x \geq 0
\]

\[
y = \frac{3}{2}x^4
\]

\[
x = \frac{3}{2}y^4
\]

\[
\frac{2}{3}x = y^4
\]

\[
x + 2 = y^3
\]

\[
\pm \sqrt{\frac{2x}{3}} = y
\]

\[
\sqrt[3]{x + 2} = y
\]

\[
\pm \sqrt[3]{\frac{2x \cdot 27}{3 \cdot 27}} = y
\]

\[
f^{-1}(x) = \sqrt[3]{\frac{54x}{3}}
\]

\[
f(x) = \frac{3}{4}x^5 + 5
\]

\[
y = \frac{3}{4}x^5 + 5
\]

\[
x = \frac{3}{4}y^5 + 5
\]

\[
x - 5 = \frac{3}{4}y^5
\]

\[
\frac{4}{3}y^3 - \frac{20}{3} = y^5
\]

\[
\sqrt[3]{\frac{4}{3} - \frac{20}{3}} = y
\]

\[
f^{-1}(x) = \sqrt[3]{\frac{4}{3} - \frac{20}{3}}
\]

\[
f(x) = -\frac{2}{5}x^6 + \frac{8}{x} \leq 0
\]

\[
y = -\frac{2}{5}x^6 + \frac{8}{x}
\]

\[
x = -\frac{2}{5}x^6 + \frac{8}{x}
\]

\[
x - 8 = -\frac{2}{5}x^6
\]

\[
-\frac{5}{2}x + 20 = y^6
\]

\[
\pm \sqrt[6]{\frac{5}{2}x + 20} = y
\]

\[
f^{-1}(x) = -\sqrt[6]{\frac{5}{2}x + 20}
\]

\[
f(x) = \frac{2x^3 - 6}{9}
\]

\[
y = \frac{2x^3 - 6}{9}
\]

\[
x = \frac{2x^3 - 6}{9}
\]

\[
x = \frac{9x = 2x^3 - 6}{9}
\]

\[
x = \frac{9x + 6 = 2x^3}{9}
\]

\[
\frac{9}{2}x + 3 = y^3
\]

\[
\sqrt[3]{\frac{9}{2}x + 3} = y
\]

\[
f^{-1}(x) = \sqrt[3]{\frac{9}{2}x + 3}
\]
Chapter 6, continued

43. \( f(x) = x^4 - 9, x \geq 0 \)
\[ y = x^4 - 9 \]
\[ x = y^4 - 9 \]
\[ x + 9 = y^4 \]
\[ \pm \sqrt[4]{x + 9} = y \]
\[ f^{-1}(x) = \sqrt[4]{x + 9} \]

44. a. False

b. True

45. \( f(x) = mx + b \)
\[ y = mx + b \]
\[ x = my + b \]
\[ x - b = my \]
\[ \frac{1}{m}x - \frac{b}{m} = y \]
\[ f^{-1}(x) = \frac{1}{m}x - \frac{b}{m} \]

Slope: \( \frac{1}{m} \)  
Intercept: \( -\frac{b}{m} \)

This is the graph of a line, and it is a function if \( m \neq 0 \).

Problem Solving

46. \( E = 0.81419D \)
\[ \frac{E}{0.81419} = D \]
When \( E = 250: D = \frac{250}{0.81419} \approx 307 \)
The amount that could be obtained for 250 euros was about $307.

47. a. \( I = 0.5w + 3 \)
\[ I - 3 = 0.5w \]
\[ \frac{I - 3}{0.5} = w \]
\[ 2I - 6 = w \]

b. When \( I = 6.5: w = 2(6.5) - 6 = 7 \)
The weight is 7 pounds.

48. a. \( C = \frac{9}{5}(F - 32) \)
\[ \frac{9}{5}C = F - 32 \]
\[ \frac{9}{5}C + 32 = F \]

From the inverse, you can convert temperatures from degrees Celsius \( C \) to degrees Fahrenheit \( F \).

b. When \( C = 5: F = \frac{9}{5}(5) + 32 = 9 + 32 = 41 \)
At the start of the race, the temperature was 41°F.
When \( C = -10: F = \frac{9}{5}(-10) + 32 = -18 + 32 = 14 \)
At the end of the race, the temperature was 14°F.

c. The temperature that is the same on both temperature scales is -40.

49. \[ \frac{v}{1.34} = \sqrt{\ell} \]
\[ \left(\frac{v}{1.34}\right)^2 = \ell \]
When \( v = 7.5: \ell = \left(\frac{7.5}{1.34}\right)^2 \approx 31.3 \)
A water line length of about 31.3 feet is needed to achieve a maximum speed of 7.5 knots.

50. \[ A = 0.2195h^{0.3964} \]
\[ \frac{A}{0.2195} = h^{0.3964} \]
\[ \left(\frac{A}{0.2195}\right)^{10.3964} = (h^{0.3964})^{10.3964} \]
\[ \left(\frac{A}{0.2195}\right)^2.523 = h \]
When \( A = 1.6: h = \left(\frac{1.6}{0.2195}\right)^{2.523} = (7.29)^{2.523} = 150 \)
The height of a 60 kilogram person who has a body surface area of 1.6 square meters is about 150 centimeters.

51. a. The function \( g(x) = -x \) is its own inverse because the graph of an inverse relation is a reflection of the graph of the original relation in the line \( y = x \) and the graph of an inverse relation is the same as the graph of the original relation.

Algebra 2
Worked-Out Solution Key
Chapter 6, continued

52. \[ g(x) = -x + b, \text{ where } b \text{ is any real number.} \]

53.

54.

55.

56.

57.

58. \[ 3x - 4y = 24 \]
   \[ 3(-2y + 2) - 4y = 24 \]
   \[ -6y + 6 - 4y = 24 \]
   \[ -10y = 18 \]
   \[ y = -\frac{9}{5} \]
   \[ \text{The solution is } \left( \frac{28}{5}, -\frac{9}{5} \right). \]

59. \[ 2x - 4y = 13 \]
   \[ 4x - 5y = 8 \]
   \[ \begin{align*}
   -4x + 8y &= -26 \\
   4x - 5y &= 8 \\
   3y &= -18 \\
   y &= -6
   \end{align*} \]

60. \[ 7x - 12y = -22 \]
   \[ 25x + 8y = 14 \]
   \[ \begin{align*}
   \times 2 \quad & 14x - 24y = -44 \\
   \times 3 \quad & 75x + 24y = 42 \\
   \hline
   89x &= 2 \\
   x &= -\frac{2}{89}
   \end{align*} \]

61. \[ 6x^3 - 54x = 0 \]
   \[ 6x(x^2 - 9) = 0 \]
   \[ 6x(x + 3)(x - 3) = 0 \]
   \[ x = 0, x = -3, \text{ or } x = 3 \]

62. \[ 2x^3 - 8x^2 + 8x = 0 \]
   \[ 2x(x^2 - 4x + 4) = 0 \]
   \[ 2x(x - 2)^2 = 0 \]
   \[ x = 0 \text{ or } x = 2 \]

63. \[ 16x^3 = -250 \]
   \[ 16x^3 + 250 = 0 \]
   \[ 2(8x^3 + 125) = 0 \]
   \[ 2[(2x)^3 + 5^3] = 0 \]
   \[ 2[(2x + 5)(4x^2 - 10x + 25)] = 0 \]
   \[ x = -\frac{5}{2} \text{ or } x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(4)(25)}}{2(4)} = \frac{10 \pm \sqrt{-300}}{8} = \frac{10 \pm 10\sqrt{3}i}{8} = \frac{5 \pm 5\sqrt{3}i}{4} \]

64. \[ x^3 - 3x^2 + 8x - 24 = 0 \]
   \[ x^2(x - 3) + 8(x - 3) = 0 \]
   \[ (x - 3)(x^2 + 8) = 0 \]
   \[ x = 3 \text{ or } x^2 + 8 = 0 \]
   \[ x^2 = -8 \]
   \[ x = \pm\sqrt{-8} = \pm 2\sqrt{2}i \]
   \[ x = 2\sqrt{2}i, x = -2\sqrt{2}i, \text{ or } x = 3 \]

65. \[ 4x^3 + 8x^2 - 25x - 50 = 0 \]
   \[ 4x^2(x + 2) - 25(x + 2) = 0 \]
   \[ (4x^2 - 25)(x + 2) = 0 \]
   \[ (2x + 5)(2x - 5)(x + 2) = 0 \]
   \[ x = -\frac{5}{2}, x = \frac{5}{2}, \text{ or } x = -2 \]
Chapter 6, continued

66. \( 12x^4 - 7x^2 - 45 = 0 \)
   \( (4x^2 - 9)(3x^2 + 5) = 0 \)
   \( (2x + 3)(2x - 3)(3x^2 + 5) = 0 \)
   \( x = \pm \frac{3}{2} \) or \( 3x^2 + 5 = 0 \)
   \( 3x^2 = -5 \)
   \( x^2 = -\frac{5}{3} \)
   \( x = \pm \sqrt[3]{\frac{-5}{3}} \)
   \( \sqrt[3]{\frac{-15}{3}} = \pm \sqrt[3]{15} \)
   \( x = \pm \sqrt[3]{\frac{3}{2}} \), \( x = \sqrt[3]{\frac{15}{3}} \)
   \( \) or \( x = -\sqrt[3]{\frac{15}{3}} \)

Quiz 6.3–6.4 (p. 445)

1. \( f(x) + g(x) = 4x^2 - x + 2x^2 = 6x^2 - x \)
   The function \( f \) and \( g \) each have the same domain: All real numbers. So, the domain of \( f + g \) also consists of all real numbers.

2. \( g(x) - f(x) = 2x^2 - (4x^2 - x) \)
   \( = 2x^2 - 4x^2 + x = -2x^2 + x \)
   The function \( f \) and \( g \) each have the same domain: All real numbers. So, the domain of \( f - g \) also consists of all real numbers.

3. \( f(x) \cdot g(x) = (4x^2 - x)(2x^2) = 8x^4 - 2x^3 \)
   The function \( f \) and \( g \) each have the same domain: All real numbers. So, the domain of \( f \cdot g \) also consists of all real numbers.

4. \( \frac{f(x)}{g(x)} = \frac{4x^2 - x}{2x^2} = \frac{2x^2 - x}{2x^2} = 2 - \frac{1}{2x} \)
   The function \( f \) and \( g \) each have the same domain: All real numbers. Because \( g(0) = 0 \), the domain of \( \frac{f}{g} \) is restricted to all real numbers except \( x = 0 \).

5. \( f(g(x)) = f(2x^2) = 4(2x^2)^2 - 2x^2 = 4(4x^4) - 2x^2 \)
   \( = 16x^4 - 2x^2 \)
   The domain of \( f(g(x)) \) consists of all real numbers.

6. \( g(f(x)) = g(4x^2 - x) = 2(4x^2 - x)^2 \)
   \( = 2(4x^2 - x)(4x^2 - x) \)
   \( = 2(16x^4 - 8x^3 + x^2) \)
   \( = 32x^4 - 16x^3 + 2x^2 \)
   The domain of \( g(f(x)) \) consists of all real numbers.

7. \( f(f(x)) = f(4x^2 - x) = 4(4x^2 - x)^2 - (4x^2 - x) \)
   \( = 4(4x^2 - x)(4x^2 - x) - 4x^2 + x \)
   \( = 4(16x^4 - 8x^3 + x^2) - 4x^2 + x \)
   \( = 64x^4 - 32x^3 + 4x^2 - 4x^2 + x \)
   \( = 64x^4 - 32x^3 + x \)
   The domain of \( f(f(x)) \) consists of all real numbers.

8. \( g(g(x)) = g(2x^2) = 2(2x^2)^2 = 2(4x^4) = 8x^4 \)
   The domain of \( g(g(x)) \) consists of all real numbers.

9. \( f(x) = x - 9, g(x) = x + 9 \)
   \( f(g(x)) = f(x + 9) \quad g(f(x)) = g(x - 9) \)
   \( f(x + 9) = x + 9 - 9 \quad g(x - 9) = x - 9 + 9 \)
   \( = x \checkmark \quad = x \checkmark \)

10. \( f(x) = 5x^3, g(x) = \sqrt[3]{x} \)
    \( f(g(x)) = f(\sqrt[3]{x}) \quad g(f(x)) = g(5x^3) \)
    \( = 5(x^{\frac{3}{3}}) \quad = \sqrt[3]{5x^3} \)
    \( = 5x \quad = \sqrt[3]{5x^3} \)
    \( \checkmark \quad \checkmark \)

11. \( f(x) = \frac{2}{3}x + 1, g(x) = \frac{2}{3}x + \frac{1}{6} \)
    \( f(g(x)) = f\left(\frac{2}{3}x + \frac{1}{6}\right) \quad g(f(x)) = g\left(\frac{2}{3}x + \frac{1}{4}\right) \)
    \( = \frac{2}{3}\left(\frac{2}{3}x + \frac{1}{6}\right) + \frac{1}{4} \quad = \frac{2}{3}\left(\frac{2}{3}x + \frac{1}{4}\right) + \frac{1}{6} \)
    \( = x + \frac{1}{4} + \frac{1}{4} \quad = x + \frac{1}{6} + \frac{1}{6} \)
    \( = x \checkmark \quad = x \checkmark \)

12. \( f(x) = 6x^2 + 1, x \geq 0 \), \( g(x) = \left(\frac{x - 1}{6}\right)^{\frac{1}{2}} \)
    \( f(g(x)) = f\left(\left(\frac{x - 1}{6}\right)^{\frac{1}{2}}\right) \quad g(f(x)) = g\left(6x^2 + 1\right) \)
    \( = 6\left(\left(\frac{x - 1}{6}\right)^{\frac{1}{2}}\right)^2 + 1 \quad = \left(6x^2 + 1 - 1\right)^{\frac{1}{2}} \)
    \( = 6\left(x - 1\right) + 1 \quad = \left(6x^2\right)^{\frac{1}{2}} \)
    \( = x + 1 + 1 \quad = (x)^{\frac{1}{2}} \)
    \( = x \checkmark \quad = x \checkmark \)

13. \( f(x) = -\frac{1}{3}x^5 + 5 \)
    \( y = -\frac{1}{3}x^5 + 5 \)
    \( x = -\frac{1}{3}y^5 + 5 \)
    \( x - 5 = -\frac{1}{3}y^5 \)
    \( -3x + 15 = y \)
    \( f^{-1}(x) = -3x + 15 \)

14. \( f(x) = x^2 - 16, x \geq 0 \)
    \( y = x^2 - 16 \quad x = y^2 - 16 \)
    \( x = y^2 - 16 \quad x + 16 = y^2 \)
    \( \sqrt{x} + 16 = y \)
    \( f^{-1}(x) = \sqrt{x} + 16 \)
Chapter 6, continued

15. \( f(x) = -\frac{2}{9}x^5 \)
    \( y = -\frac{2}{9}x^5 \)
    \( x = -\frac{2}{9}y^5 \)
    \( -\frac{9}{2}x = y^5 \)
    \( \sqrt{\frac{9}{2}x} = y \)
    \( \sqrt{\frac{9x + 16}{2 + 16}} = y \)
    \( \sqrt{\frac{9x + 36}{3}} = y \)
    \( f^{-1}(x) = -\sqrt{\frac{9x + 36}{3}} \)

16. \( f(x) = 5x + 12 \)
    \( y = 5x + 12 \)
    \( x = 5y + 12 \)
    \( x - 12 = 5y \)
    \( \frac{1}{5}x - \frac{12}{5} = y \)
    \( f^{-1}(x) = \frac{1}{5}x - \frac{12}{5} \)

17. \( f(x) = -3x^3 - 4 \)
    \( y = -3x^3 - 4 \)
    \( x = -3y^3 \)
    \( x + 4 = -3y^3 \)
    \( \frac{x + 4}{-3} = y^3 \)
    \( \sqrt[3]{x + 4} \)
    \( \sqrt[3]{-3 \cdot 9} = y \)
    \( \sqrt[3]{9x + 36} \)
    \( \frac{3}{y} \)
    \( f^{-1}(x) = \sqrt[3]{\frac{9x + 36}{3}} \)

18. \( f(x) = 9x^4 - 49, x \leq 0 \)
    \( y = 9x^4 - 49 \)
    \( x = 9y^4 - 49 \)
    \( x + 49 = 9y^4 \)
    \( \frac{x + 49}{9} = y^4 \)
    \( \pm \sqrt[3]{\frac{x + 49}{9}} = y \)
    \( \pm \sqrt[3]{\frac{x + 49 \cdot 9}{9 \cdot 9}} = y \)
    \( \pm \frac{\sqrt[3]{9x + 441}}{3} = y \)
    \( f^{-1}(x) = -\frac{\sqrt[3]{9x + 441}}{3} \)

19. \( C(g(d)) = C(0.02d) = 2.15(0.02d) = 0.043d \)
    \( C(g(400)) = 0.043(400) = 17.2 \)
    The expression \( C(g(400)) \) or $17.20 represents the cost of gasoline for the car that is driven 400 miles.

Lesson 6.5

6.5 Guided Practice (pp. 447–449)

1. \( y = -3\sqrt{x} \)

2. \( f(x) = \frac{1}{2}\sqrt{x} \)

3. \( y = -\frac{1}{2}\sqrt{x} \)

Chapter 6, continued
Chapter 6, continued

4. \( g(x) = 4\sqrt{x} \)

\[
\begin{array}{c|c|c|c|c}
 x & -2 & -1 & 0 & 1 \\
 f(x) & -5.04 & -4 & 4 & 5.04 \\
\end{array}
\]

The domain and range are all real numbers.

5. A pendulum with a period of 1 second is about 0.81 foot long.

6. \( y = -4\sqrt{x} + 2 \)

Because \( h = 0 \) and \( k = 2 \), shift the graph of \( y = -4\sqrt{x} \) up 2 units.

The domain is \( x \geq 0 \) and the range is \( y \leq 2 \).

7. \( y = 2\sqrt{x} + 1 \)

Because \( h = 1 \) and \( k = 0 \), shift the graph of \( y = 2\sqrt{x} \) left 1 unit.

The domain is \( x \geq -1 \) and the range is \( y \geq 0 \).

8. \( f(x) = \frac{1}{2}\sqrt{x} - 3 - 1 \)

Because \( h = 3 \) and \( k = -1 \), shift the graph of \( f(x) = \frac{1}{2}\sqrt{x} \) right 3 units and down 1 unit.

The domain is \( x \geq 3 \) and the range is \( f(x) \geq -1 \).

9. \( y = 2\sqrt{x} - 4 \)

Because \( h = 4 \) and \( k = 0 \), shift the graph of \( y = 2\sqrt{x} \) right 4 units.

The domain and the range are both all real numbers.

10. \( y = \sqrt{x} - 5 \)

Because \( h = 0 \) and \( k = -5 \), shift the graph of \( y \sqrt{x} \) down 5 units.

The domain and range are both all real numbers.

11. \( g(x) = -\sqrt{x} + 2 - 3 \)

Because \( h = -2 \) and \( k = -3 \), shift the graph of \( g(x) = -\sqrt{x} \) left 2 units and down 3 units.

The domain and range are both all real numbers.

6.5 Exercises (pp. 449–451)

Skill Practice

1. Square root functions and cube root functions are examples of radical functions.

2. a. When \( a = -3 \), the graph of \( y = \sqrt{x} \) is stretched vertically by a factor of 3 and then reflected in the \( x \)-axis.
   
   b. When \( h = 2 \), the graph of \( y = \sqrt{x} \) is shifted to the right 2 units.
   
   c. When \( k = 4 \), the graph of \( y = \sqrt{x} \) is shifted up 4 units.
Chapter 6, continued

3. \( y = -4\sqrt[2]{x} \)

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0</td>
<td>-4</td>
<td>-5.66</td>
<td>-6.93</td>
<td>-8</td>
</tr>
</tbody>
</table>

The domain is \( x \geq 0 \) and the range is \( y \leq 0 \).

4. \( f(x) = \frac{1}{2}\sqrt[3]{x} \)

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>0</td>
<td>0.5</td>
<td>0.71</td>
<td>0.87</td>
<td>1</td>
</tr>
</tbody>
</table>

The domain is \( x \geq 0 \) and the range is \( f(x) \geq 0 \).

5. \( y = -\sqrt[3]{\frac{2}{3}x} \)

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0</td>
<td>-0.8</td>
<td>-1.13</td>
<td>-1.39</td>
<td>-1.6</td>
</tr>
</tbody>
</table>

The domain is \( x \geq 0 \) and the range is \( y \leq 0 \).

6. \( y = -6\sqrt[3]{x} \)

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0</td>
<td>-6</td>
<td>-8.49</td>
<td>-10.39</td>
<td>-12</td>
</tr>
</tbody>
</table>

The domain is \( x \geq 0 \) and the range is \( y \leq 0 \).

7. \( y = 5\sqrt[3]{x} \)

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0</td>
<td>5</td>
<td>7.07</td>
<td>8.66</td>
<td>10</td>
</tr>
</tbody>
</table>

The domain is \( x \geq 0 \) and the range is \( y \geq 0 \).

8. \( g(x) = 9\sqrt[3]{x} \)

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>g(x)</td>
<td>0</td>
<td>9</td>
<td>12.73</td>
<td>15.59</td>
<td>18</td>
</tr>
</tbody>
</table>

The domain is \( x \geq 0 \) and the range is \( g(x) \geq 0 \).

9. \( D; \ y = -\frac{3}{2}\sqrt[3]{x} \)

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0</td>
<td>-1.5</td>
<td>-2.12</td>
<td>-2.6</td>
<td>-3</td>
</tr>
</tbody>
</table>

The domain is \( x \geq 0 \) and the range is \( y \leq 0 \).

10. \( y = \frac{1}{4}\sqrt[3]{x} \)

<table>
<thead>
<tr>
<th>x</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-0.32</td>
<td>-0.25</td>
<td>0</td>
<td>0.25</td>
<td>0.31</td>
</tr>
</tbody>
</table>

The domain and range are all real numbers.
Chapter 6, continued

11. \( y = 2\sqrt{x} \)

<table>
<thead>
<tr>
<th>x</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-2.52</td>
<td>-2</td>
<td>0</td>
<td>2</td>
<td>2.52</td>
</tr>
</tbody>
</table>

The domain and range are all real numbers.

12. \( f(x) = -5\sqrt{x} \)

<table>
<thead>
<tr>
<th>x</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>6.3</td>
<td>5</td>
<td>0</td>
<td>-5</td>
<td>-6.3</td>
</tr>
</tbody>
</table>

The domain and range are all real numbers.

13. \( h(x) = -\frac{1}{2}\sqrt{x} \)

<table>
<thead>
<tr>
<th>x</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>h(x)</td>
<td>0.18</td>
<td>0.14</td>
<td>0</td>
<td>-0.14</td>
<td>-0.18</td>
</tr>
</tbody>
</table>

The domain and range are all real numbers.

14. \( g(x) = 6\sqrt{x} \)

<table>
<thead>
<tr>
<th>x</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>g(x)</td>
<td>-7.56</td>
<td>-6</td>
<td>6</td>
<td>6</td>
<td>7.56</td>
</tr>
</tbody>
</table>

The domain and range are all real numbers.

15. \( y = \frac{7}{9}\sqrt{x} \)

<table>
<thead>
<tr>
<th>x</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>h(x)</td>
<td>-0.98</td>
<td>-0.78</td>
<td>0</td>
<td>0.78</td>
<td>0.98</td>
</tr>
</tbody>
</table>

The domain and range are all real numbers.

16. \( f(x) = 2\sqrt{x} - 1 + 3 \)

Because \( h = 1 \) and \( k = 3 \), shift the graph of \( f(x) = 2\sqrt{x} \) right 1 unit and up 3 units.

The domain is \( x \geq 1 \) and the range is \( f(x) \geq 3 \).

17. \( y = (x + 1)^{1/2} + 8 = \sqrt{x + 1} + 8 \)

Because \( h = -1 \) and \( k = 8 \), shift the graph of \( y = x^{1/2} \) left 1 unit and up 8 units.

The domain is \( x \geq -1 \) and the range is \( y \geq 8 \).
Chapter 6, continued

18. \( y = -4\sqrt{x - 5} + 1 \)

Because \( h = 5 \) and \( k = 1 \), shift the graph of \( y = -4\sqrt{x} \) right 5 units and up 1 unit.

The domain is \( x \geq 5 \) and the range is \( y \leq 1 \).

19. \( y = \frac{3}{4^{1/3}} - 1 = \frac{3\sqrt[3]{4}}{4} - 1 \)

Because \( h = 0 \) and \( k = -1 \), shift the graph of \( y = \frac{3\sqrt[3]{4}}{4}x - 1 \) down 1 unit.

The domain and range are both all real numbers.

20. \( y = -2\sqrt{x} + 5 + 5 \)

Because \( h = -5 \) and \( k = 5 \), shift the graph of \( y = -2\sqrt{x} \) left 5 units and up 5 units.

The domain and range are both all real numbers.

21. \( h(x) = -3\sqrt{x} + 7 - 6 \)

Because \( h = -7 \) and \( k = -6 \), shift the graph of \( h(x) = -3\sqrt{x} \) left 7 units and down 6 units.

The domain and range are both all real numbers.

22. \( y = -\sqrt{x - 4} - 7 \)

Because \( h = 4 \) and \( k = -7 \), shift the graph of \( y = -\sqrt{x} \) right 4 units and down 7 units.

The domain is \( x \geq 4 \) and the range is \( y \leq -7 \).

23. \( g(x) = \frac{1}{3}\sqrt{x} - 6 \)

Because \( h = 0 \) and \( k = -6 \), shift the graph of \( g(x) = \frac{1}{3}\sqrt{x} \) down 6 units.

The domain and range are both all real numbers.

24. \( y = 4\sqrt{x} - 4 + 5 \)

Because \( h = 4 \) and \( k = 5 \), shift the graph of \( y = 4\sqrt{x} \) right 4 units and up 5 units.

The domain and range are both all real numbers.

25. The domain of \( y = \sqrt{x - 5} + 4 \) is limited because of the square root of a negative number is not a real number.

Because the domain is restricted, the range is also affected.

26. The error occurred in describing the horizontal translation.

Because \( h = -1 \), the graph is translated left 1 unit. So, graph of \( y = -2\sqrt{x + 1} - 3 \) is the graph of \( y = -2\sqrt{x} \) translated left 1 unit and down 3 units.

27. \( C \);

Because the graph of \( y = 3\sqrt{x} \) is shifted left 2 units, \( h = -2 \) and \( k = 0 \). An equation of the translated graph is \( y = 3\sqrt{x + 2} \).

28. The function \( y = \sqrt{x + 5} \) is a translation of the function \( y = \sqrt{x} \) that has a domain of \( x \geq 0 \) and a range of \( y \geq 0 \).

Because \( h = -5 \), the domain is \( x \geq -5 \). Because \( k = 0 \), the range is \( y \geq 0 \).
Chapter 6, continued

29. The function \( y = \sqrt{x} - 12 \) is a translation of the function \( y = \sqrt{x} \) that has a domain \( x \geq 0 \) and a range of \( y \geq 0 \). Because \( h = -12 \), the domain is \( x \geq 12 \). Because \( k = 0 \), the range is \( y \geq 0 \).

30. The function \( y = \frac{1}{2} \sqrt{x} - 4 \) is a translation of the function \( y = \frac{1}{2} \sqrt{x} \) that has a domain of \( x \geq 0 \) and a range of \( y \geq 0 \). Because \( h = -4 \), the domain is \( x \geq 0 \). Because \( k = 0 \), the range is \( y \geq 0 \).

31. The function \( y = \frac{1}{2} \sqrt{x} + 7 \) is a translation of the function \( y = \frac{1}{2} \sqrt{x} \) that has a domain of all real numbers. Therefore, the function \( y = \frac{1}{2} \sqrt{x} + 7 \) also has a domain and range of all real numbers.

32. The function \( g(x) = \sqrt{x} + 7 \) is a translation of the function \( g(x) = \sqrt{x} \) that has a domain of all real numbers. Therefore, the function \( g(x) = \sqrt{x} + 7 \) also has a domain and range of all real numbers.

33. The function \( f(x) = \frac{1}{2} \sqrt{x} - 3 + 6 \) is a translation of the function \( f(x) = \frac{1}{4} \sqrt{x} \) that has a domain of \( x \geq 0 \) and a range of \( f(x) \geq 0 \). Because \( h = 3 \), the domain is \( x \geq 3 \). Because \( k = 6 \), the range is \( f(x) \geq 6 \).

34. When \( n \) is even, the domain of the function \( y = \sqrt{x} \) is \( x \geq 0 \) and the range of the function is \( y \geq 0 \). When \( n \) is odd, the domain and the range of the function \( y = \sqrt{x} \) are both all real numbers.

Problem Solving

35. You can see 8 miles at an altitude of about 43 feet above sea level.

36. a. A pendulum with a length of 2 feet has a period of about 1.6 seconds.

37. a. \[ v = 331.5 \sqrt{\frac{K}{273.15}}, \quad K \geq 0 \]
\[ = 331.5 \sqrt{\frac{273.15 + C}{273.15}} \]
\[ = 331.5 \sqrt{1 + \frac{C}{273.15}} \]

b. Because \( K \geq 0 \) and \( K = 273.15 + C \):
\[ 273.15 + C \geq 0 \]
\[ C \geq -273.15 \]
The domain of the function is \( C \geq -273.15 \).
The range of the function is \( v \geq 0 \).

38. The power of a 3500 pound car that reaches a speed of 200 miles per hour is about 2500 horsepower.

39. a. \[ v_i = 33.7 \sqrt{\frac{W}{A}} = 33.7 \sqrt{\frac{165}{A}} \]

b. | \( A \) | 2 | 4 | 6 | 8 | 10 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( v_i )</td>
<td>306.1</td>
<td>216.44</td>
<td>176.72</td>
<td>153.05</td>
<td>136.89</td>
</tr>
</tbody>
</table>
Chapter 6, continued

40. a. 
\[ s = \pi r + \pi r^2 \]
\[ s = \pi (r + r^2) \]
\[ s + \pi \left(\frac{1}{4}\right) = \pi \left(r^2 + r + \frac{1}{4}\right) \]
\[ s + \frac{\pi}{4} = \pi (r + \frac{1}{2})^2 \]
\[ \frac{1}{\pi} s + \frac{\pi}{4} = r + \frac{1}{2} \]
\[ \frac{1}{\pi} \sqrt{s + \frac{\pi}{4}} = r + \frac{1}{2} \]
\[ \frac{1}{\pi} \sqrt{s + \frac{\pi}{4} - \frac{1}{2}} = r \]

b. 

44. \[ x^2 - 14x + 37 = 0 \]
\[ x^2 - 14x + 49 = -37 + 49 \]
\[ (x - 7)^2 = 12 \]
\[ x - 7 = \pm 2\sqrt{3} \]
\[ x = 7 \pm 2\sqrt{3} \]

45. \[ x^2 - 40x + 383 = 0 \]
\[ x^2 - 40x + 400 = -383 + 400 \]
\[ (x - 20)^2 = 17 \]
\[ x - 20 = \pm \sqrt{17} \]
\[ x = 20 \pm \sqrt{17} \]

46. \[ x^2 - 22x + 97 = 0 \]
\[ x^2 - 22x = -97 \]
\[ x^2 - 22x + 121 = -97 + 121 \]
\[ (x - 11)^2 = 24 \]
\[ x - 11 = \pm 2\sqrt{6} \]
\[ x = 11 \pm 2\sqrt{6} \]

47. \[ f(x) = x^4 + 5x^3 - 3x^2 - 9x \]
\[ = x(x^3 + 5x^2 - 3x - 9) \]
Using factoring and the zero (or root) feature of a graphing calculator, the zeros are \( x = 0, \) \( x = -5.2, \) \( x = -1.2, \) and \( x = 1.4. \)

48. \[ f(x) = x^4 - 7x^3 - 4x^2 - 8x + 22 \]
The possible rational zeros are \( \pm 1, \pm 2, \pm 11, \) and \( \pm 22. \)
Because none of these satisfy \( f(x) = 0, \) there are no rational zeros.

Using the zero (or root) feature of a graphing calculator, the real zeros are \( x \approx 1.1 \) and \( x \approx 7.6. \) The two remaining zeros are complex conjugates. One factor of the polynomial can be approximated as \( (x - 7.6)(x - 1.1) = x^2 - 8.76x + 8.36. \)

Using long division:
\[ \frac{x^3 - 7x^3 - 4x^2 - 8x + 22}{x^2 - 8.76x + 8.36} = x^2 + 1.7x + 2.43 \]
So, \( x = \frac{-1.7 \pm \sqrt{1.7^2 - 4(1)(2.43)}}{2(1)} \)
\[ = -0.85 \pm \frac{\sqrt{-6.83}}{2} = -0.85 \pm 1.3i \]
So, the zeros of the function are \( x = 1.1, x = 7.6, \) and \( x = -0.85 \pm 1.3i. \)
### 6.6 Guided Practice (pp. 452–455)

1. \( \sqrt[3]{x} = -9 \)  
   \( \sqrt[3]{-9} = 8 \)  
   \( \left( \sqrt[3]{-9} \right)^2 = 8^2 \)  
   \( x = 512 \)  
   Check: \( \sqrt[3]{512} - 9 \not= -1 \)  
   \[ \begin{align*}  
   8 - 9 & \not= -1 \\
   -1 & = -1 \checkmark
   \end{align*} \]

2. \( \sqrt{x + 25} = 4 \)  
   \( \left( \sqrt{x + 25} \right)^2 = 4^2 \)  
   \( x + 25 = 16 \)  
   \( x = -9 \)  
   Check: \( \sqrt{-9 + 25} \not= 4 \)  
   \( \sqrt{16} \not= 4 \)  
   \( 4 = 4 \checkmark \)

3. \( 2\sqrt{x - 3} = 4 \)  
   \( \left( 2\sqrt{x - 3} \right)^2 = 4^2 \)  
   \( x - 3 = 8 \)  
   \( x = 11 \)  
   Check: \( 2\sqrt{11 - 3} \not= 4 \)  
   \( 2\sqrt{8} \not= 4 \)  
   \( 4 = 4 \checkmark \)

4. \( v(p) = 6.3\sqrt{1013 - p} \)  
   \( 48.3 = 6.3\sqrt{1013 - p} \)  
   \( 7.67 = \sqrt{1013 - p} \)  
   \( 7.67^2 = \left( \sqrt{1013 - p} \right)^2 \)  
   \( 58.8 = 1013 - p \)  
   \( -954.2 = -p \)  
   \( 954.2 = p \)  
   The air pressure at the center of the hurricane is about 954 millibars.

5. \( 3x^{3/2} = 375 \)  
   \( x^{3/2} = 125 \)  
   \( (x^{3/2})^{2/3} = 125^{2/3} \)  
   \( x = 25 \)  
   Check: \( 3(25)^{3/2} \not= 375 \)  
   \( 3(125) \not= 375 \)  
   \( 375 = 375 \checkmark \)

6. \( -2x^{3/4} = -16 \)  
   \( x^{3/4} = 8 \)  
   \( (x^{3/4})^{4/3} = 8^{4/3} \)  
   \( x = 16 \)  
   Check: \( -2(16)^{3/4} \not= -16 \)  
   \(-2(8) \not= -16 \)  
   \(-16 = -16 \checkmark \)

7. \( -\frac{2}{3}x^{1/5} = -2 \)  
   \( x^{1/5} = 3 \)  
   \( (x^{1/5})^5 = 3^5 \)  
   \( x = 243 \)  
   Check: \( -\frac{2}{3}(243)^{1/5} \not= -2 \)  
   \( \frac{2}{3}(3) \not= -2 \)  
   \(-2 = -2 \checkmark \)
Chapter 6, continued

8. \((x + 3)^{5/2} = 32\)
   \[\frac{(x + 3)^{5/2}}{2} = \frac{32}{2}\]
   \[x + 3 = 4\]
   \[x = 1\]
   Check: \(1 + 3)^{5/2} = 32\)

9. \((x - 5)^{4/3} = 81\)
   \[\frac{(x - 5)^{4/3}}{3} = \frac{81^{3/4}}{3}\]
   \[x - 5 = \pm(81^{3/4})\]
   \[x - 5 = \pm(3^3)\]
   \[x = 5\]
   \[x = 5 \pm 27\]
   \[x = 32\] or \(x = -22\)
   Check \(x = 32:\)
   \(32 - 5)^{4/3} = 81\)

10. \((x + 2)^{2/3} + 3 = 7\)
    \([x + 2)^{2/3} = 4\]
    \[\frac{(x + 2)^{2/3}}{3} = \frac{4^{3/2}}{3}\]
    \[x + 2 = (4^{3/2})\]
    \[x + 2 = \pm(2^3)\]
    \[x + 2 = \pm 8\]
    \[x = 6\] or \(x = -10\)
    Check \(x = 6:\)
    \(6 + 2)^{2/3} + 3 = 7\)

11. \(x - \frac{1}{2} = \frac{1}{\sqrt{4}}\)
    \[\left(x - \frac{1}{2}\right)^2 = \left(\frac{1}{\sqrt{4}}\right)^2\]
    \[x^2 - x + \frac{1}{4} = \frac{1}{4}\]
    \[x^2 - \frac{5}{4} + \frac{1}{4} = 0\]
    \[(x - 1)(x - 1) = 0\]
    \[x = 1\] or \(x = \frac{1}{4}\)
    Check \(x = 1:\)
    \(1 - \frac{1}{2} = \sqrt{\frac{1}{4}}\)
    \[\frac{1}{4} - \frac{1}{2} \neq \frac{1}{4}\]

12. \(\sqrt{10x + 9} = x + 3\)
    \[(\sqrt{10x + 9})^2 = (x + 3)^2\]
    \[10x + 9 = x^2 + 6x + 9\]
    \[0 = x^2 - 4x\]
    \[0 = x(x - 4)\]
    \[x = 0\] or \(x = 4\)
    Check \(x = 0:\)
    \(\sqrt{10(0) + 9} = 0 + 3\)
    \(\sqrt{9} = 3\)
    \[3 = 3\]
    \(\sqrt{10(4) + 9} = 4 + 3\)
    \(\sqrt{49} = 7\)
    \[7 = 7\]
    The solutions are 0 and 4.

13. \(\sqrt{2x + 5} = \sqrt{x + 7}\)
    \[(\sqrt{2x + 5})^2 = (\sqrt{x + 7})^2\]
    \[2x + 5 = x + 7\]
    \[x = 2\]
    Check: \(\sqrt{2(2) + 5} = \sqrt{2 + 7}\)
    \[3 = 3\]

14. \(\sqrt{x + 6} - 2 = \sqrt{x - 2}\)
    \[(\sqrt{x + 6} - 2)^2 = (\sqrt{x - 2})^2\]
    \[x + 6 - 4\sqrt{x + 6} + 4 = x - 2\]
    \[-4\sqrt{x + 6} = -12\]
    \[\sqrt{x + 6} = 3\]
    \[(\sqrt{x + 6})^2 = 3^2\]
    \[x + 6 = 9\]
    \[x = 3\]
    Check: \(\sqrt{3 + 6} - 2 = \sqrt{3 - 2}\)
    \[\sqrt{9} - 2 = \sqrt{1}\]
    \[3 - 2 = 1\]
    \[1 = 1\]

6.6 Exercises (pp. 456–459)

Skill Practice

1. When you solve an equation algebraically, an apparent solution that must be rejected because it does not satisfy the original equation is called an extraneous solution.

2. To avoid this solution, the student could have added \(\sqrt{9x - 5}\) to each side of the equation before squaring each side.

3. \(\sqrt{5x + 1} = 6\)
    \[(\sqrt{5x + 1})^2 = 6^2\]
    \[5x + 1 = 36\]
    \[5x = 35\]
    \[x = 7\]
    Check: \(\sqrt{5(7) + 1} = 6\)
    \[\sqrt{36} = 6\]
    \[6 = 6\]
Chapter 6, continued

4. \( \sqrt{3x+10} = 8 \)
   \( (\sqrt{3x+10})^2 = 8^2 \)
   \( 3x + 10 = 64 \)
   \( 3x = 54 \)
   \( x = 18 \)
   Check: \( \sqrt{3(18)} + 10 \neq 8 \)
   \( \sqrt{64} \neq 8 \)
   \( 8 = 8 \checkmark \)

5. \( \sqrt{9x} + 11 = 14 \)
   \( \sqrt{9x} = 3 \)
   \( (\sqrt{9x})^2 = 3^2 \)
   \( 9x = 9 \)
   \( x = 1 \)
   Check: \( \sqrt{9(1)} + 11 \neq 14 \)
   \( \sqrt{9} + 11 \neq 14 \)
   \( 14 = 14 \checkmark \)

6. \( \sqrt{2x} - \frac{2}{3} = 0 \)
   \( \sqrt{2x} = \frac{2}{3} \)
   \( (\sqrt{2x})^2 = \left(\frac{2}{3}\right)^2 \)
   \( 2x = \frac{4}{9} \)
   \( x = \frac{2}{9} \)
   Check: \( \left[\sqrt{2\left(\frac{2}{9}\right)} - \frac{2}{3}\right] = 0 \)
   \( \sqrt{\frac{4}{9}} - \frac{2}{3} = 0 \)
   \( 0 = 0 \checkmark \)

7. \( -2\sqrt{24x} + 13 = -11 \)
   \( -2\sqrt{24x} = -24 \)
   \( \sqrt{24x} = 12 \)
   \( (\sqrt{24x})^2 = 12^2 \)
   \( 24x = 144 \)
   \( x = 6 \)
   Check: \( -2\sqrt{24(6)} + 13 \neq -11 \)
   \( -2\sqrt{144} + 13 \neq -11 \)
   \( -2(12) + 13 \neq -11 \)
   \( -11 = -11 \checkmark \)

8. \( 8\sqrt{10x} - 7 = 9 \)
   \( 8\sqrt{10x} = 16 \)
   \( \sqrt{10x} = 2 \)
   \( (\sqrt{10x})^2 = 2^2 \)
   \( 10x = 4 \)
   \( x = \frac{2}{5} \)
   Check: \( 8\left(\sqrt{10\left(\frac{2}{5}\right)} - 7 \right) \neq 9 \)
   \( 8(2) - 7 \neq 9 \)
   \( 9 = 9 \checkmark \)

9. \( \sqrt{x - 25} = 3 \)
   \( (\sqrt{x - 25})^2 = 3^2 \)
   \( x - 25 = 9 \)
   \( x = 34 \)
   Check: \( \sqrt{34 - 25} = 3 \)
   \( \sqrt{9} = 3 \)
   \( 3 = 3 \checkmark \)

10. \( -4\sqrt{x} - 6 = -20 \)
    \( -4\sqrt{x} = -14 \)
    \( \sqrt{x} = \frac{7}{2} \)
    \( (\sqrt{x})^2 = \left(\frac{7}{2}\right)^2 \)
    \( x = \frac{49}{4} \)
    Check: \( -4\sqrt{\frac{49}{4}} - 6 \neq -20 \)
    \( -4\left(\frac{7}{2}\right) - 6 \neq -20 \)
    \( -14 - 6 \neq -20 \)
    \( -20 = -20 \)

11. \( \sqrt{2x + 3} = 10 \)
    \( (\sqrt{2x + 3})^2 = 10^2 \)
    \( 2x + 3 = 100 \)
    \( 2x = 97 \)
    \( x = \frac{97}{2} \)
    Check: \( \sqrt{2\left(\frac{97}{2}\right) + 3} \neq 10 \)
    \( \sqrt{99 + 3} \neq 10 \)
    \( 10 = 10 \checkmark \)

12. \( C: \)
    \( \sqrt{8x + 3} = 3 \)
    \( (\sqrt{8x + 3})^2 = 3^2 \)
    \( 8x + 3 = 9 \)
    \( 8x = 6 \)
    \( x = \frac{3}{4} \)
    Check: \( \sqrt{8\left(\frac{3}{4}\right) + 3} \neq 3 \)
    \( \sqrt{6 + 3} \neq 3 \)
    \( \sqrt{9} \neq 3 \)
    \( 3 = 3 \checkmark \)
Chapter 6, continued

13. $\sqrt{x} - 10 = -3$
   $\sqrt{x} = 7$
   $(\sqrt{x})^2 = 7^2$
   $x = 49$
   Check: $\sqrt{49} - 10 \neq -3$
   $7 - 10 \neq -3$
   $-3 = -3 \checkmark$

14. $\sqrt{x} - 16 = 2$
   $(\sqrt{x} - 16)^2 = 2^2$
   $x - 16 = 8$
   $x = 24$
   Check: $\sqrt{24} - 16 \neq 2$
   $\frac{2}{8} \neq 2$
   $2 = 2 \checkmark$

15. $\sqrt{12}x - 13 = -7$
   $\sqrt{12}x = 6$
   $(\sqrt{12})^2 = 6^2$
   $12x = 216$
   $x = 18$
   Check: $\sqrt{12}(18) - 13 \neq -7$
   $\sqrt{216} - 13 \neq -7$
   $6 - 13 \neq -7$
   $-7 = -7 \checkmark$

16. $3\sqrt{16}x - 7 = 17$
   $3\sqrt{16}x = 24$
   $\sqrt{16}x = 8$
   $(\sqrt{16})^2 = 8^2$
   $16x = 512$
   $x = 32$
   Check: $3\sqrt{16}(32) - 7 \neq 17$
   $3\sqrt{512} - 7 \neq 17$
   $24 - 7 \neq 17$
   $17 = 17 \checkmark$

17. $-5\sqrt{8}x + 12 = -8$
   $-5\sqrt{8}x = -20$
   $\sqrt{8}x = 4$
   $(\sqrt{8})^2 = 4^2$
   $8x = 64$
   $x = 8$
   Check: $-5\sqrt{8}(8) + 12 \neq -8$
   $-5\sqrt{64} + 12 \neq -8$
   $-20 + 12 \neq -8$
   $-8 = -8 \checkmark$

18. $\sqrt{4x} + 5 = \frac{1}{2}$
   $(\sqrt{4x} + 5)^2 = \left(\frac{1}{2}\right)^2$
   $4x + 5 = \frac{1}{4}$
   $4x = -\frac{19}{4}$
   $x = -\frac{19}{16}$
   Check: $\sqrt{4\left(-\frac{19}{16}\right)} + 5 \neq \frac{1}{2}$
   $\sqrt{-\frac{19}{4}} + 5 \neq \frac{1}{2}$
   $\frac{\sqrt{19}}{2} \neq \frac{1}{2}$
   $\frac{1}{2} = \frac{1}{2} \checkmark$

19. $\sqrt{x} - 3 + 2 = 4$
   $\sqrt{x} - 3 = 2$
   $(\sqrt{x} - 3)^2 = 2^2$
   $x - 3 = 8$
   $x = 11$
   Check: $\sqrt{11} - 3 + 2 \neq 4$
   $\sqrt{8} + 2 \neq 4$
   $4 = 4 \checkmark$

20. $\sqrt{4x} + 2 - 6 = -10$
   $\sqrt{4x} + 2 = -4$
   $(\sqrt{4x} + 2)^2 = (-4)^2$
   $4x + 2 = -64$
   $4x = -66$
   $x = -\frac{33}{2}$
   Check: $\sqrt{4\left(-\frac{33}{2}\right)} + 2 - 6 \neq -10$
   $\sqrt{-66} + 2 - 6 \neq -10$
   $\sqrt{-66} - 4 \neq -10$
   $-4 = -10 \checkmark$

21. $-4\sqrt{x} + 10 + 3 = 15$
   $-4\sqrt{x} + 10 = 12$
   $\sqrt{x} + 10 = -3$
   $(\sqrt{x} + 10)^2 = (-3)^2$
   $x + 10 = -27$
   $x = -37$
   Check: $-4\sqrt{-37} + 10 + 3 \neq 15$
   $-4\sqrt{-27} + 3 \neq 15$
   $12 + 3 \neq 15$
   $15 = 15 \checkmark$
Chapter 6, continued

22. Sample answer: \( \sqrt[3]{-3x - 1} = 2 \)

Substitute \(-3\) for \(x\) in the equation \( \sqrt[3]{ax + b} = c \) and find values of \(a, b,\) and \(c\) that satisfy the equation. Let \(c = 2\).

\[
\sqrt[3]{a(-3)} + b = 2
\]
\[
\sqrt[3]{8} = 2
\]
\[-3a + b = 8
\]
\[-3(-3) - 1 = 8
\]
\[a = -3, b = -1, c = 2
\]

23. \(2x^{2/3} = 32\)

\(x^{2/3} = 16\)

\((x^{2/3})^{3/2} = 16^{3/2}\)

\[x = \pm(16^{3/2})^3
\]

\[x = \pm64
\]

Check \(x = 64:\)

\[2(64)^{2/3} \not\simeq 32
\]

\[2 (16)^{2/3} \not\simeq 32
\]

\[32 = 32 \checkmark
\]

Check \(x = -64:\)

\[2(-64)^{2/3} \not\simeq 32
\]

\[2(16)^{2/3} \not\simeq 32
\]

\[32 = 32 \checkmark
\]

24. \(\frac{1}{2}x^{5/2} = 16\)

\(x^{5/2} = 32\)

\((x^{5/2})^{2/5} = 32^{2/5}\)

\[x = 4
\]

Check: \(\frac{1}{2}(4^{5/2})^2 \not\simeq 16\)

\[\frac{1}{2}(32)^2 \not\simeq 16
\]

\[16 = 16 \checkmark
\]

25. \(9x^{2/5} = 36\)

\(x^{2/5} = 4\)

\((x^{2/5})^{5/2} = 4^{5/2}\)

\[x = \pm(4^{5/2})^5
\]

\[x = \pm256
\]

Check \(x = 32:\)

\[9(32)^{2/5} \not\simeq 36
\]

\[9(32) \not\simeq 36
\]

\[9(4) \not\simeq 36
\]

\[36 = 36 \checkmark
\]

Check \(x = -32:\)

\[9(-32)^{2/5} \not\simeq 36
\]

\[9(-32) \not\simeq 36
\]

\[9(4) \not\simeq 36
\]

\[36 = 36 \checkmark
\]

26. \((8x)^{4/3} + 44 = 300\)

\((8x)^{4/3} = 256\)

\[[(8x)^{4/3}]^{3/4} = 256^{3/4}\]

\[8x = \pm(256^{1/4})^3
\]

\[8x = \pm(256)^{3/4}
\]

\[8x = \pm(4)^3
\]

\[8x = \pm64
\]

\[x = \pm8
\]

Check \(x = 8:\)

\[[8(8)^{4/3} + 44 = 300
\]

\[64^{4/3} + 44 \not\simeq 300
\]

\[256 + 44 \not\simeq 300
\]

\[300 = 300 \checkmark
\]

Check \(x = -8:\)

\[[8(-8)^{4/3} + 44 = 300
\]

\[(-64)^{4/3} + 44 \not\simeq 300
\]

\[256 + 44 \not\simeq 300
\]

\[300 = 300 \checkmark
\]

27. \(\frac{1}{2}(x + 9)^{3/2} = 49\)

\((x + 9)^{3/2} = 343\)

\[[(x + 9)^{3/2}]^{2/3} = 343^{2/3}\]

\[x + 9 = 49
\]

\[x = 40
\]

Check: \((40 + 9)^{3/2} \not\simeq 343\)

\[49^{3/2} \not\simeq 343
\]

\[343 = 343 \checkmark
\]

28. \((x - 5)^{5/3} - 73 = 170\)

\((x - 5)^{5/3} = 243\)

\[[(x - 5)^{5/3}]^{3/5} = 243^{3/5}\]

\[x - 5 = 27
\]

\[x = 32
\]

Check: \((32 - 5)^{5/3} - 73 \not\simeq 170\)

\[27^{5/3} - 73 \not\simeq 170
\]

\[243 - 73 = 170
\]

\[170 = 170 \checkmark
\]

29. \(\left[\frac{1}{3}x - 11\right]^{1/2} = 5\)

\[\left[\frac{1}{3}x - 11\right]^{1/2} = 5\]

\[
\left[\frac{1}{3}x - 11\right]^{1/2} = 5
\]

\[
\frac{1}{3}x = 36
\]

\[x = 108
\]

Check: \(\left[\frac{1}{3}(108) - 11\right]^{1/2} \not\simeq 5\)

\[(36 - 11)^{1/2} \not\simeq 5
\]

\[25^{1/2} \not\simeq 5
\]

\[5 = 5 \checkmark
\]

30. \((5x - 19)^{5/6} = 32\)

\[[(5x - 19)^{5/6}]^{6/5} = 32^{6/5}\]

\[5x - 19 = 64
\]

\[5x = 83
\]

\[x = \frac{83}{5}
\]

Check: \(\left[\frac{83}{5} - 19\right]^{5/6} \not\simeq 32\)

\[(83 - 19)^{5/6} \not\simeq 32
\]

\[64^{5/6} \not\simeq 32
\]

\[32 = 32 \checkmark
\]
Chapter 6, continued

31. \((3x + 43)^{2/3} + 22 = 38\)
   \((3x + 43)^{2/3} = 16\)
   \(\left[(3x + 43)^{2/3}\right]^{1/2} = 16^{1/2}\)
   \(3x + 43 = \pm (16^{1/2})^3\)
   \(3x + 43 = \pm (4)^3\)
   \(3x + 43 = \pm 64\)
   \(3x = -43 \pm 64\)
   \(x = \frac{-107}{3}\)

Check \(x\) = 7:
\(3\left(\frac{-107}{3}\right)^{2/3} + 22 \approx 38\)
\(64^{2/3} + 22 \approx 38\)
\(16 + 22 \approx 38\)
\(38 = 38 \checkmark\)

32. Before each side of the equation was cubed, the radical should have been isolated on the one side of the equation.
   \(\sqrt[3]{x} + 2 = 4\)
   \(\sqrt[3]{x} = 2\)
   \(\left(\sqrt[3]{x}\right)^3 = 2^3\)
   \(x = 8\)

33. Each side of the equation should have been squared, no just one side.
   \((x + 7)^{1/2} = 5\)
   \(\left[(x + 7)^{1/2}\right]^2 = 5^2\)
   \(x + 7 = 25\)
   \(x = 18\)

34. \(x - 6 = \sqrt{3x}\)
   \((x - 6)^2 = (\sqrt{3x})^2\)
   \(x^2 - 12x + 36 = 3x\)
   \(x^2 - 15x + 36 = 0\)
   \((x - 12)(x - 3) = 0\)
   \(x - 12 = 0\) or \(x - 3 = 0\)
   \(x = 12\) or \(x = 3\)

Check \(x = 12:\)
\(12 - 6 \div \sqrt[3]{12} \approx 36\)
\(6 = 6 \checkmark\)

The only solution is 12.

35. \(x - 10 = \sqrt{9x}\)
   \((x - 10)^2 = (\sqrt{9x})^2\)
   \(x^2 - 20x + 100 = 9x\)
   \(x^2 - 29x + 100 = 0\)
   \((x - 4)(x - 25) = 0\)
   \(x - 4 = 0\) or \(x - 25 = 0\)
   \(x = 4\) or \(x = 25\)

Check \(x = 4:\)
\(4 - 10 \div \sqrt{9(4)}
\(-6 \div \sqrt{36}\)
\(-6 \div 6\)

The only solution is 25.

36. \(x = \sqrt{16x + 225}\)
   \(x^2 = (\sqrt{16x + 225})^2\)
   \(x^2 = 16x + 225\)
   \(x^2 - 16x - 225 = 0\)
   \((x - 25)(x + 9) = 0\)
   \(x - 25 = 0\) or \(x + 9 = 0\)
   \(x = 25\) or \(x = -9\)

Check \(x = 25:\)
\(25 \div \sqrt{16(25) + 225}\)
\(-9 \div \sqrt{16(-9) + 225}\)

The only solution is 25.

37. \(\sqrt{21x + 1} = x + 5\)
   \((\sqrt{21x + 1})^2 = (x + 5)^2\)
   \(21x + 1 = x^2 + 10x + 25\)
   \(0 = x^2 - 11x + 24\)
   \(0 = (x - 8)(x - 3)\)
   \(x - 8 = 0\) or \(x - 3 = 0\)
   \(x = 8\) or \(x = 3\)

Check \(x = 8:\)
\(\sqrt{21(8) + 1} \div 8 + 5\)
\(\sqrt{169} \div 13\)
\(13 = 13 \checkmark\)

The solutions are 3 and 8.
Chapter 6, continued

38. \( \sqrt{44 - 2x} = x - 10 \)
   \((\sqrt{44 - 2x})^2 = (x - 10)^2 \)
   
   \(44 - 2x = x^2 - 20x + 100 \)
   
   \(0 = x^2 - 18x + 56 \)
   
   \(0 = (x - 4)(x - 14) \)
   
   \(x - 4 = 0 \quad \text{or} \quad x - 14 = 0 \)
   
   \(x = 4 \quad \text{or} \quad x = 14 \)
   
   Check \( x = 4 \): \( \sqrt{44 - 2(4)} \neq 4 - 10 \)
   
   \(\sqrt{36} \neq -6 \)
   
   \(6 \neq 6 \)
   
   The only solution is 14.

39. \( \sqrt{x^2 + 4} = x + 5 \)
   
   \((\sqrt{x^2 + 4})^2 = (x + 5)^2 \)
   
   \(x^2 + 4 = x^2 + 10x + 25 \)
   
   \(-21 = 10x \)
   
   \(-21 \neq 10x \)
   
   Check: \( \sqrt{\left(\frac{-21}{10}\right)^2 + 4} \neq \frac{21}{10} + 5 \)
   
   \(\sqrt{8.41} \neq 2.9 \)
   
   \(2.9 \neq 2.9 \)
   
   The only solution is 14.

40. \( x - 2 = \sqrt{\frac{3}{2}x - 2} \)
   
   \((x - 2)^2 = \left(\sqrt{\frac{3}{2}x - 2}\right)^2 \)
   
   \(x^2 - 4x + 4 = \frac{3}{2}x - 2 \)
   
   \(x^2 - \frac{11}{2}x + 6 = 0 \)
   
   \((x - 4)(x - \frac{3}{2}) = 0 \)
   
   \(x - 4 = 0 \quad \text{or} \quad x - \frac{3}{2} = 0 \)
   
   \(x = 4 \quad \text{or} \quad x = \frac{3}{2} \)
   
   Check \( x = 4 \): \( 4 - 2 \neq \sqrt{\frac{3}{2}(4) - 2} \)
   
   \(2 \neq \sqrt{4} \)
   
   \(2 \neq 2 \)
   
   The only solution is 4.
Chapter 6, continued

42. \(\sqrt{8x^3 - 1} = 2x - 1\)

\((\sqrt{8x^3 - 1})^2 = (2x - 1)^2\)

\(8x^3 - 1 = (2x - 1)(2x - 1)(2x - 1)\)

\(8x^3 - 1 = (4x^2 - 4x + 1)(2x - 1)\)

\(8x^3 - 1 = 8x^3 - 8x^2 + 2x - 4x^2 + 4x - 1\)

\(12x^2 - 6x = 0\)

\(6x(2x - 1) = 0\)

\(6x = 0\) or \(2x - 1 = 0\)

\(x = 0\) or \(x = \frac{1}{2}\)

Check \(x = 0\): \(\sqrt{8(0)^3 - 1} = \sqrt{8(0) - 1} = \sqrt{7}\)

\(\sqrt{7} \neq 1\)

\(-1 = -1\)

\(\sqrt{0} = 0\)

The solutions are 0 and \(\frac{1}{2}\).

43. \(\sqrt{32x - 64} = 2x\)

\((\sqrt{32x - 64})^2 = (2x)^2\)

\(32x - 64 = 4x^2\)

\(0 = 4x^2 - 32x + 64\)

\(0 = 4(x^2 - 8x + 16)\)

\(0 = 4(x - 4)(x - 4)\)

\(x - 4 = 0\)

\(x = 4\)

You can tell that \(\sqrt{x + 4} = -5\) has no solution because the positive square root of a number is never negative.

45. \(\sqrt{4x + 1} = \sqrt{x + 10}\)

\((\sqrt{4x + 1})^2 = (\sqrt{x + 10})^2\)

\(4x + 1 = x + 10\)

\(3x = 9\)

\(x = 3\)

Check: \(\sqrt{4(3) + 1} = \sqrt{3 + 10}\)

\(\sqrt{13} = \sqrt{13}\)

46. \(\sqrt{12x - 5} - \sqrt{8x + 15} = 0\)

\((\sqrt{12x - 5} - \sqrt{8x + 15})^2 = (\sqrt{12x - 5})^2 - 2\sqrt{12x - 5}\sqrt{8x + 15} + (\sqrt{8x + 15})^2\)

\(12x - 5 = 8x + 15\)

\(4x = 20\)

\(x = 5\)

Check: \(\sqrt{12(5) - 5} - \sqrt{8(5) + 15} = 0\)

\(\sqrt{55} - \sqrt{55} = 0\)

\(0 = 0\)

47. \(\sqrt{3x - 8} + 1 = \sqrt{x + 5}\)

\((\sqrt{3x - 8} + 1)^2 = (\sqrt{x + 5})^2\)

\(3x - 8 + 2\sqrt{3x - 8} + 1 = x + 5\)

\(2\sqrt{3x - 8} = -x + 6\)

\((\sqrt{3x - 8})^2 = (-x + 6)^2\)

\(3x - 8 = x^2 - 12x + 36\)

\(0 = x^2 - 15x + 44\)

\(0 = (x - 4)(x - 11)\)

\(x - 4 = 0\) or \(x - 11 = 0\)

\(x = 4\) or \(x = 11\)

Check: \(x = 4:\)

\(\sqrt{3(4) - 8 + 1} \neq \sqrt{4 + 5}\)

\(\sqrt{3(11) - 8 + 1} \neq \sqrt{11 + 5}\)

\(3 = 3\)

\(6 \neq 4\)

The only solution is 4.

48. \(\frac{2}{3}x^4 - 4 = \frac{2}{3}x^3 - 7\)

\(\frac{2}{3}x^4 - 4 = \left(\frac{2}{3}x^3 - 7\right)^2\)

\(\frac{2}{3}x^4 - 4 = \frac{2}{3}x^3 - 7\)

\(\frac{4}{3}x = -3\)

\(x = -\frac{45}{4}\)

Check: \(\frac{2}{3} \cdot \left(-\frac{45}{4}\right)^4 - 4 = \left(\frac{2}{3} \cdot \frac{-45}{4}\right)^3 - 7\)

\(\frac{15}{2} - 4 = \frac{9}{2} - 7\)

\(\frac{23}{2} = \frac{23}{2}\)

49. \(\sqrt{x + 2} = 2 - \sqrt{x}\)

\((\sqrt{x + 2})^2 = (2 - \sqrt{x})^2\)

\(x + 2 = 4 - 4\sqrt{x} + x\)

\(4\sqrt{x} = 2\)

\(\sqrt{x} = 1\)

\((\sqrt{x})^2 = \frac{1}{2}\)

\(x = \frac{1}{4}\)

Check: \(\frac{1}{4} + 2 = 2 - \sqrt{\frac{1}{4}}\)

\(\frac{9}{4} \neq 2 - \frac{1}{2}\)

\(\frac{3}{2} = \frac{3}{2}\)

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Algebra 2

398 Worked-Out Solution Key
Chapter 6, continued

50. \( \sqrt{2x + 3} + 2 = \sqrt{6x + 7} \)
\((\sqrt{2x + 3} + 2)^2 = (\sqrt{6x + 7})^2 \)
\(2x + 3 + 4\sqrt{2x + 3} + 4 = 6x + 7 \)
\(4\sqrt{2x + 3} = 4x \)
\(\sqrt{2x + 3} = x \)
\((\sqrt{2x + 3})^2 = x^2 \)
\(2x + 3 = x^2 \)
\(0 = x^2 - 2x - 3 \)
\(0 = (x - 3)(x + 1) \)
\(x - 3 = 0 \quad \text{or} \quad x + 1 = 0 \)
\(x = 3 \quad \text{or} \quad x = -1 \)

Check \(x = 3:\)
\(\sqrt{2(3)} + 3 + 2 = \sqrt{6(3)} + 7 \)
\(\sqrt{9} + 2 = \sqrt{25} \)
\(5 = 5 \checkmark \)

Check \(x = -1:\)
\(\sqrt{2(-1)} + 3 + 2 = \sqrt{6(-1)} + 7 \)
\(\sqrt{1} + 2 = \sqrt{1} \)
\(3 = 3 \checkmark \)

The only solution is 3.

51. \( \sqrt{2x + 5} = \sqrt{x + 2} + 1 \)
\((\sqrt{2x + 5})^2 = (\sqrt{x + 2} + 1)^2 \)
\(2x + 5 = x + 2 + 2\sqrt{x + 2} + 1 \)
\(x + 2 = 2\sqrt{x + 2} \)
\((x + 2)^2 = (2\sqrt{x + 2})^2 \)
\(x^2 + 4x + 4 = 4(x + 2) \)
\(x^2 + 4x + 4 = 4x + 8 \)
\(x^2 = 4 \)
\(x = \pm\sqrt{4} = \pm 2 \)

Check \(x = 2:\)
\(\sqrt{2(2)} + 5 = \sqrt{2} + 2 + 1 \)
\(\sqrt{9} = \sqrt{2} + 1 \)
\(3 = 3 \checkmark \)

Check \(x = -2:\)
\(\sqrt{2(-2)} + 5 = \sqrt{-2} + 2 + 1 \)
\(\sqrt{1} = \sqrt{0} + 1 \)
\(1 = 1 \checkmark \)

The solutions are 2 and -2.

52. \( \sqrt{5x + 6} + 3 = \sqrt{3x + 3} + 4 \)
\(\sqrt{5x + 6} = \sqrt{3x + 3} + 1 \)
\((\sqrt{5x + 6})^2 = (\sqrt{3x + 3} + 1)^2 \)
\(5x + 6 = 3x + 3 + 2\sqrt{3x + 3} + 1 \)
\(2x + 2 = 2\sqrt{3x + 3} \)
\(x + 1 = \sqrt{3x + 3} \)
\((x + 1)^2 = (\sqrt{3x + 3})^2 \)
\(x^2 + 2x + 1 = 3x + 3 \)
\(x^2 - x - 2 = 0 \)
\((x - 2)(x + 1) = 0 \)
\(x - 2 = 0 \quad \text{or} \quad x + 1 = 0 \)
\(x = 2 \quad \text{or} \quad x = -1 \)

Check \(x = 2:\)
\(\sqrt{5(2)} + 3 = \sqrt{3(2)} + 3 + 4 \)
\(\sqrt{16} + 3 = \sqrt{9} + 4 \)
\(5 = 7 \checkmark \)

Check \(x = -1:\)
\(\sqrt{5(-1)} + 3 = \sqrt{3(-1)} + 3 + 4 \)
\(\sqrt{1} + 3 = \sqrt{0} + 4 \)
\(4 = 4 \checkmark \)

The solutions are 2 and -1.

53. \( 3\sqrt{x} + 5\sqrt{y} = 31 \)
\(5\sqrt{x} - 5\sqrt{y} = -15 \)
\(8\sqrt{x} = 16 \)
\(\sqrt{x} = 2 \)
\((\sqrt{x})^2 = 2^2 \)
\(x = 4 \)
\(3\sqrt{4} + 5\sqrt{y} = 31 \)
\(6 + 5\sqrt{y} = 31 \)
\(5\sqrt{y} = 25 \)
\(\sqrt{y} = 5 \)
\((\sqrt{y})^2 = 5^2 \)
\(y = 25 \)

The solution is (4, 25).

54. \( 5\sqrt{x} - 2\sqrt{y} = 4\sqrt{2} \)
\(2\sqrt{x} + 3\sqrt{y} = 13\sqrt{2} \)
\(15\sqrt{x} - 6\sqrt{y} = 12\sqrt{2} \)
\(4\sqrt{x} + 6\sqrt{y} = 26\sqrt{2} \)
\(19\sqrt{x} = 38\sqrt{2} \)
\(\sqrt{x} = 2\sqrt{2} \)
\((\sqrt{x})^2 = (2\sqrt{2})^2 \)
\(x = 8 \)

\(5\sqrt{8} - 2\sqrt{y} = 4\sqrt{2} \)
\(10\sqrt{2} - 2\sqrt{y} = 4\sqrt{2} \)
\(-2\sqrt{y} = -6\sqrt{2} \)
\(\sqrt{y} = 3\sqrt{2} \)
\((\sqrt{y})^2 = (3\sqrt{2})^2 \)
\(y = 18 \)

The solution is (8, 18).
Chapter 6, continued

55. Sample answer: \( \sqrt{4x - 3} = -x \)
   \((\sqrt{4x - 3})^2 = (-x)^2 \)
   \(4x - 3 = x^2 \)
   \(0 = x^2 - 4x + 3 \)
   \(0 = (x - 1)(x - 3) \)
   \(x = 1 \) or \( x = 3 \)

Check \( x = 1: \)
\( \sqrt{4(1) - 3} \neq -1 \)
\( \sqrt{1} \neq -1 \)
\( 1 \neq -1 \)

Check \( x = 3: \)
\( \sqrt{4(3) - 3} \neq -3 \)
\( \sqrt{9} \neq -3 \)
\( 3 \neq -3 \)

There is no solution.

Problem Solving

56. \( v = \sqrt{2gh} \)
   \( 15 = \sqrt{2(9.8)}h \)
   \( 15 = \sqrt{19.6}h \)
   \( 15^2 = (\sqrt{19.6}h)^2 \)
   \( 225 = 19.6h \)
   \( 11.5 \approx h \)

The height at the top of the swing was about 11.5 meters.

57. \( r = \frac{kl}{\pi(b_0 - h)} \)
   \( 0.875 = \frac{0.04l}{\pi(6.5 - 0)} \)
   \( 0.875 = \frac{0.04l}{\pi b_0} \)
   \( 0.875 = \frac{0.04l}{\pi b_0} \)
   \( 4 = \frac{0.04l}{\pi b_0} \)
   \( 4^2 = (\frac{0.04l}{\pi b_0})^2 \)
   \( 16 = \frac{0.04l}{\pi b_0} \)
   \( 400 = l \)

It will take about 400 minutes for the entire candle to burn.

58. \( l = 54a^{3/2} \)
   \( 3 = 54a^{3/2} \)
   \( \frac{1}{18} = a^{3/2} \)
   \( \left( \frac{1}{18} \right)^{2/3} = (a^{3/2})^{2/3} \)
   \( 0.15 = a \)

The diameter of the nail is about 0.15 inch.

59. \( h = 62.5\sqrt{t} + 75.8 \)

Elephant with \( h = 150: \) \( 150 = 62.5\sqrt{t} + 75.8 \)
\( 74.2 = 62.5\sqrt{t} \)
\( 1.2 = \sqrt{t} \)
\( 1.44 = (\sqrt{t})^2 \)
\( 2 = t \)

Elephant with \( h = 250: \) \( 250 = 62.5\sqrt{t} + 75.8 \)
\( 174.2 = 62.5\sqrt{t} \)
\( 2.7 = \sqrt{t} \)
\( 7.29 = (\sqrt{t})^2 \)
\( 22 = t \)

The elephant with a shoulder height of 250 centimeter is about 20 years older than the elephant with a shoulder height of 150 centimeters.

60. a. \( t = 0.5\sqrt{h} \)

Basketball player: Kangaroo:
\( 0.81 = 0.5\sqrt{h} \)
\( 1.62 = \sqrt{h} \)
\( 1.62^2 = (\sqrt{h})^2 \)
\( 2.6 = h \)
\( 5 = h \)

The basketball player jumped a height of about 2.6 feet.

b. Basketball player: Kangaroo:
\( 2(0.81) = 0.5\sqrt{h} \)
\( 3.24 = \sqrt{h} \)
\( 3.24^2 = (\sqrt{h})^2 \)
\( 10.5 = h \)
\( 4.82 = (\sqrt{h})^2 \)
\( 20 = h \)

c. No; if the hang time doubles, the height jumped does not double, it quadruples. To find the new height jumped, you should double the hang time and square the results, which means that the new height will be \( 2^3 = 4 \) times greater than the old height.

61. a. \( B = 1.69\sqrt{s} + 4.25 - 3.55 \)
   \( 0 = 1.69\sqrt{s} + 4.25 - 3.55 \)
   \( 3.55 = 1.69\sqrt{s} + 4.25 \)
   \( 2.12 = (\sqrt{s} + 4.25)^2 \)
   \( 4.41 = s + 4.25 \)
   \( 0.16 = s \)

The windspeed is about 0.16 mile per hour.

b. \( B = 1.69\sqrt{s} + 4.25 - 3.55 \)
   \( 12 = 1.69\sqrt{s} + 4.25 - 3.55 \)
   \( 15.55 = 1.69\sqrt{s} + 4.25 \)
   \( 9.2 = \sqrt{s} + 4.25 \)
   \( 9.2^2 = (\sqrt{s} + 4.25)^2 \)
   \( 84.64 = s + 4.25 \)
   \( 80.39 = s \)

The windspeed is about 80.39 miles per hour.

c. \( 0.16 \leq s \leq 80.39 \)
Chapter 6, continued

62. \[ l = \sqrt{h^2 + \frac{1}{4}(b_2 - b_1)^2} \]
   
   \[ 5 = \sqrt{h^2 + \frac{1}{4}(4 - 2)^2} \]
   
   \[ 5 = \sqrt{h^2 + 1} \]
   
   \[ 5^2 = (\sqrt{h^2 + 1})^2 \]
   
   \[ 25 = h^2 + 1 \]
   
   \[ 24 = h^2 \]
   
   \[ \sqrt{24} = h \]
   
   \[ 2\sqrt{6} = h \]
   
   \[ 4.9 = h \]

   The height of the pyramid is 4.9 units.

Mixed Review

63. \[ 4^3 \cdot 4^2 = 4^{3+2} \] Product of powers property
   
   \[ = 4^5 \]
   
   \[ = 1024 \]
   
64. \[ (3^{-2})^3 = 3^{-2\cdot3} \] Power of a power property
   
   \[ = 3^{-6} \]
   
   \[ = \frac{1}{3^6} \]
   
   \[ = \frac{1}{729} \]

65. \[ (-5)(-5)^{-4} = (-5)^1(-5)^{-4} \] Product of powers property
   
   \[ = (-5)^{-3} \]
   
   \[ = \frac{1}{(-5)^3} \] Negative exponent property
   
   \[ = \frac{1}{125} \]

66. \[ (10^{-3})^{-1} = 10^{-3\cdot(-1)} \] Power of a power property
   
   \[ = 10^3 \]
   
   \[ = 1000 \]

67. \[ 8^{-4} \cdot 8^3 = 8^{-4+3} \] Product of powers property
   
   \[ = 8^{-1} \]
   
   \[ = \frac{1}{8} \] Negative exponent property

68. \[ 6^0 \cdot 6^4 \cdot 6^{-4} = 6^{0+4+(-4)} \] Product of powers property
   
   \[ = 6^0 \]
   
   \[ = 1 \] Zero exponent property

69. \[ f(x) = -x^3 \]

<table>
<thead>
<tr>
<th>x</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>27</td>
<td>8</td>
<td>1</td>
<td>0</td>
<td>-8</td>
<td>-27</td>
<td></td>
</tr>
</tbody>
</table>

70. \[ f(x) = x^4 - 9 \]

<table>
<thead>
<tr>
<th>x</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>72</td>
<td>7</td>
<td>-8</td>
<td>-9</td>
<td>-8</td>
<td>7</td>
<td>72</td>
</tr>
</tbody>
</table>

71. \[ f(x) = x^3 + 2 \]

<table>
<thead>
<tr>
<th>x</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
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</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>-25</td>
<td>-6</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>10</td>
<td>29</td>
</tr>
</tbody>
</table>

72. \[ f(x) = x^4 - 8x^2 - 48 \]

<table>
<thead>
<tr>
<th>x</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>-39</td>
<td>-64</td>
<td>-55</td>
<td>-48</td>
<td>-55</td>
<td>-64</td>
<td>-39</td>
</tr>
</tbody>
</table>

73. \[ f(x) = -\frac{1}{3}x^3 + x \]

<table>
<thead>
<tr>
<th>x</th>
<th>-6</th>
<th>-3</th>
<th>0</th>
<th>3</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>66</td>
<td>6</td>
<td>0</td>
<td>-6</td>
<td>-66</td>
</tr>
</tbody>
</table>
Chapter 6, continued

74. \( f(x) = x^2 - 2x - 4 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>-32</td>
<td>-3</td>
<td>-4</td>
<td>-5</td>
<td>24</td>
</tr>
</tbody>
</table>

75. \( 16^{3/2} = (\sqrt{16})^3 = 4^3 = 64 \)

76. \( \frac{1}{8^{2/3}} = 8^{2/3} = (\sqrt[3]{8})^2 = 2^2 = 32 \)

77. \( -256^{1/4} = -2^{5/4} = -\sqrt[4]{2^5} = -\sqrt[4]{32} = -2 \)

78. \( 4^{-5/2} = \frac{1}{4^{5/2}} = \frac{1}{(\sqrt{4})^5} = \frac{1}{2^5} = \frac{1}{32} \)

79. \( 3125^{-\frac{2}{5}} = \frac{1}{3125^{2/5}} = \frac{1}{(\sqrt[5]{3125})^2} = \frac{1}{5^2} = \frac{1}{25} \)

80. \( \frac{1}{27^{1/3}} = \frac{1}{(\sqrt[3]{27})} = \frac{1}{3} = \frac{1}{81} \)

Quiz 6.5–6.6 (p. 459)

1. \( y = 4\sqrt{x} \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>0</td>
<td>4</td>
<td>5.66</td>
<td>6.93</td>
<td>8</td>
</tr>
</tbody>
</table>

2. \( y = \sqrt{x} + 3 \)

Because \( h = 0 \) and \( k = 3 \), shift the graph of \( y = \sqrt{x} \) up 3 units.

The domain is \( x \geq 0 \) and range is \( y \geq 3 \).

3. \( g(x) = \sqrt{x} + 2 - 5 \)

Because \( h = -2 \) and \( k = -5 \), shift the graph of \( g(x) = \sqrt{x} \) left 2 units and down 5 units.

The domain is \( x \geq -2 \) and range is \( y \geq -5 \).

4. \( y = -\frac{1}{2}\sqrt{x} \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>0.63</td>
<td>0.5</td>
<td>0</td>
<td>-0.5</td>
<td>-0.63</td>
</tr>
</tbody>
</table>

The domain and range are both all real numbers.

5. \( f(x) = \sqrt[3]{x} - 4 \)

The domain and range are both all real numbers.

6. \( y = \sqrt[3]{x} - 3 + 2 \)

Because \( h = 3 \) and \( k = 2 \), shift the graph of \( y = \sqrt[3]{x} \) to the right 3 units and up 2 units. The domain and range are both all real numbers.

7. \( \sqrt{6x + 15} = 9 \)

\( (\sqrt{6x + 15})^2 = 9^2 \)

\( 6x + 15 = 81 \)

\( 6x = 66 \)

\( x = 11 \)

Check: \( \sqrt{6(11) + 15} \geq 9 \)

\( \sqrt{81} = 9 \)

\( 9 = 9 \checkmark \)
Chapter 6, continued

8. \( \frac{1}{4}(7x + 8)^{3/2} = 54 \)
\[(7x + 8)^{3/2} = 216 \]
\[7x + 8 = 36 \]
\[x = 28 \]

Check: \( \frac{1}{4}(7(4) + 8)^{3/2} \neq 54 \)

9. \( \sqrt[4]{3}x + 5 + 2 = 5 \)
\( \sqrt[4]{3}x + 5 = 3 \)
\( (\sqrt[4]{3}x + 5)^3 = 3^3 \)
\[3x + 5 = 27 \]
\[3x = 22 \]
\[x = \frac{22}{3} \]

Check: \( \sqrt[4]{3}(\frac{22}{3}) + 5 + 2 \neq 5 \)

10. \( x - 3 = \sqrt{10x - 54} \)
\[x - 3 = \sqrt{(\sqrt{10x} - 54)} \]
\[x^2 - 6x + 9 = 10x - 54 \]
\[x^2 - 16x + 63 = 0 \]
\[(x - 9)(x - 7) = 0 \]
\[x = 9 \] or \[x = 7 \]

Check \( x = 9 \): \( 9 - 3 = \sqrt{10(9) - 54} \)
\[6 = \sqrt{36} \]
\[6 = 6 \checkmark \]

The solutions are 7 and 9.

11. \( \sqrt{4x - 4} = \sqrt{5}x - 1 - 1 \)
\[(\sqrt{4x - 4})^2 = (\sqrt{5}x - 1 - 1)^2 \]
\[4x - 4 = 5x - 1 - 2\sqrt{5}x - 1 + 1 \]
\[2\sqrt{5}x - 1 = x + 4 \]
\[(2\sqrt{5}x - 1)^2 = (x + 4)^2 \]
\[4(5x - 1) = x^2 + 8x + 16 \]
\[20x - 4 = x^2 + 8x + 16 \]
\[0 = x^2 - 12x + 20 \]
\[0 = (x - 10)(x - 2) \]
\[x - 10 = 0 \] or \[x - 2 = 0 \]
\[x = 10 \] or \[x = 2 \]

Check \( x = 10 \):
\[\sqrt{4(10)} - 4 \neq \sqrt{5(10)} - 1 - 1 \]
\[\sqrt{36} \neq \sqrt{49} - 1 \]
\[6 = 6 \checkmark \]

Check \( x = 2 \):
\[\sqrt{4(2)} - 4 \neq \sqrt{5(2)} - 1 - 1 \]
\[\sqrt{4} \neq \sqrt{9} - 1 \]
\[2 = 2 \checkmark \]

The solutions are 2 and 10.

12. \( \sqrt[4]{5}x - 9 = \sqrt{x - 6} \)
\[\left( \sqrt[4]{5}x - 9 \right)^3 = (\sqrt{x - 6})^3 \]
\[\frac{4}{5}x - 9 = x - 6 \]
\[-3 = \frac{1}{5}x \]
\[-15 = x \]

Check: \( \sqrt[4]{\frac{4}{5}(-15)} - 9 \neq \sqrt{-15 - 6} \)
\[\sqrt{-21} \neq \sqrt{-21} \checkmark \]

The solution is -15.

13. \( P = 0.199a^{3/2} \)
\[(1.88 \text{ years}) \left( \frac{365 \text{ days}}{1 \text{ year}} \right) = 686.2 \text{ days} \]
\[686.2 = 0.199a^{3/2} \]
\[3448 = a^{3/2} \]
\[3448^{2/3} = (a^{3/2})^{2/3} \]
\[228 = a \]

The length of the orbit’s minor axis is about 228 million kilometers.

Problem Solving Workshop 6.6 (p. 461)

1. \( \sqrt{25} - x = 8 \)

The solution is -39.
Chapter 6, continued

2. $2.3\sqrt{x} - 1 = 11.5$

The solution is $x = 26$.

3. $4.3\sqrt{x} - 7 = 30$

The solution is about $x = 55.7$.

4. $6\sqrt{2} - 7x - 1.2 = 22.8$

The solution is $x = -2$.

5. $d = \sqrt{625 + h^2}$

$s = \sqrt{625 + h^2}$

The rocket’s height is about 97 feet.

6. $w(p) = 6.3\sqrt{1013 - p}$

The mean sustained wind velocity is 25 meters per second when the air pressure is about 997 millibars.

7. $L = \pi r\sqrt{r^2 + h^2}$

$900 = \pi(7.5)\sqrt{(7.5)^2 + h^2}$

$900 = 7.5\pi \sqrt{56.25 + h^2}$

The height of the cone is about 37 centimeters.

6.6 Extension (p.463)

1. $2\sqrt{x} - 5 \geq 3$

The solution is $x \geq 2$

2. $\sqrt{x} - 4 \leq 5$

The solution of the inequality is $x \leq 16$.

3. $4\sqrt{x} + 1 \leq 9$

The domain is $x \geq 0$.

4. $\sqrt{x} + 7 \geq 3$

The domain is $x \geq -7$.

The solution of the inequality is $x \geq 2$. 
Chapter 6, continued

5. \(\sqrt{x} + \sqrt{x + 3} \geq 3\)

The domain is \(x \geq 0\).
The solution of the inequality is \(x \geq 1\).

6. \(\sqrt{x} + \sqrt{x - 5} \leq 5\)

The domain is \(x \geq 5\).
The solution of the inequality is \(5 \leq x \leq 25\).

7. \(2\sqrt{x} + 3 \leq 8\)

The domain of \(y = 2\sqrt{x} + 3\) is \(x \geq 0\).
The solution of the inequality is \(0 \leq x \leq 2.25\).

8. \(\sqrt{x} + 3 \geq 2.6\)

The domain is \(x \geq -3\).
The solution of the inequality is \(x \geq 0.25\).

9. \(7\sqrt{x} + 1 < 9\)

The domain of \(y = 7\sqrt{x} + 1\) is \(x \geq 0\).
The solution of the inequality is approximately \(0 \leq x < 1.3\).

10. \(4\sqrt{3x} - 7 > 7.8\)

The domain of \(y = 4\sqrt{3x} - 7\) is \(x \geq \frac{7}{3}\).
The solution of the inequality is \(x > 3.6\).

11. \(\sqrt{x} - \sqrt{x + 5} < -1\)

The domain of \(y = \sqrt{x} - \sqrt{x + 5}\) is \(x \geq 0\).
The solution of the inequality is \(0 \leq x < 4\).

12. \(\sqrt{x} + 2 + \sqrt{x - 1} \leq 9\)

The domain of \(y = \sqrt{x} + 2 + \sqrt{x - 1}\) is \(x \geq 1\).
The solution of the inequality is approximately \(1 \leq x \leq 19.8\).

13. \(\frac{1}{2} + 1.25\sqrt{s} - 9.8\sqrt{d} \leq 16\)

The domain of \(y = \frac{1}{2} + 1.25\sqrt{s} - 9.8\sqrt{d}\) is \(s \geq 0\).
The solution of the inequality is approximately \(0 \leq s \leq 413\). The maximum allowable value for \(s\) is about 413 square meters.
Chapter 6, continued

Mixed Review of Problem Solving (p. 464)

1. a. \( R = C + MC \)
   \[ R = C + 0.4C \]
   \[ R = 1.4C \]

   b. \( R = 1.4C \)

   \[ \frac{R}{1.4} = C \]

   c. \( C = \frac{R}{1.4} = \frac{60}{1.4} = 42.86 \)

   The cost of a pair of the women's athletic shoes is $42.86.

2. \( \sqrt{3x - 5} = 4 \)
   \( (\sqrt{3x - 5})^2 = 4^2 \)
   \[ 3x - 5 = 16 \]
   \[ 3x = 21 \]
   \[ x = 7 \]

   You can use the graph of \( y = \sqrt{3x - 5} \) and the graph of \( y = 4 \) to verify that \( x = 7 \) is correct by finding the \( x \)-coordinate value of their intersection point.

3. Sample answer: \( \sqrt{3x + 19} = 2 \)

   Working Backwards:
   \[ x = -5 \]
   \[ 3x = -15 \]
   \[ 3x + 19 = 4 \]
   \[ \sqrt{3x + 19} = 2 \]

4. a. \( k = 0.134d \)

   \[ \frac{k}{0.134} = d \]

   The inverse function is \( d = \frac{k}{0.134} \).

   b. \( d = \frac{25}{0.134} = 186.57 \)

   You receive $186.57 for 25 kronor.

   c. The function \( d = \frac{k}{0.134} \) gives U.S. dollar in terms of Swedish kronor.

5. \( y = \sqrt{x} + 14 \)
   \( (14, 0): 0 = \sqrt{-14 + 14} \)
   \( (-13, 1): 1 = \sqrt{-13 + 14} \)

   Yes, there are other square root functions that pass through these points; two points do not determine a unique square root function.

6. Sample answer: \( y = 3\sqrt{x} - 4, y = -\sqrt{x} - 2 \)

7. a. \( v = 8V \bar{h} \)

   \[ \frac{v}{8} = \sqrt{h} \]

   \[ \left( \frac{v}{8} \right)^2 = \left( \sqrt{h} \right)^2 \]

   \[ \frac{v^2}{64} = h \]

   b. Sample answer: What is the maximum height of a toy rocket launched upward from ground level that has an initial velocity of 45 feet per second?

   \[ h = \frac{v^2}{64} = \frac{45^2}{64} = \frac{2025}{64} = 31.6 \]

   The maximum height of the toy rocket is 31.6 feet.

8. \( d = \sqrt{2500 + h^2} \)
   \[ 100 = \sqrt{2500 + h^2} \]
   \[ 100^2 = (\sqrt{2500 + h^2})^2 \]
   \[ 10,000 = 2500 + h^2 \]
   \[ h^2 = 2500 \]
   \[ \pm \sqrt{2500} = h \]
   \[ \pm 50 = h \]

   Reject the negative value. The height of the balloon is about 50 feet.

9. \( r(t) = 6t, A(r) = \pi r^2 \)

   \[ A(r(2)) = \pi (4)^2 = 16\pi \]

   The composition \( A(r(2)) \), or 452 square feet, represents the area of the outer ripple formed 2 seconds after the pebble hits the water.

Chapter 6 Review (pp. 466–468)

1. The index of the radical \( \sqrt[3]{27} \) is 3.

2. Sample answer: \( \sqrt[6]{6}, 8\sqrt[6]{6}; \frac{1}{2}\sqrt{x}, -\frac{3}{5}\sqrt{x} \)

3. Sample answer: A power function has the form \( y = ax^b \) where \( a \) is a real number and \( b \) is a rational number.

4. The graph of a function’s inverse is a reflection of the graph of the original function in the line \( y = x \).

5. The inverse of a function \( f \) is also a function if and only if no horizontal line intersects the graph of \( f \) more than once.

6. The graph of \( y = \sqrt[5]{x} - 4 + 5 \) is a shift of the graph of \( y = \sqrt[5]{x} \) units right and 5 units up.

7. \( x^{2/3} = 5 \)

   \( (x^{2/3})^3 = 5^3 \)

   \[ x^2 = 125 \]

   The student will have to take the square root of each side. To solve the equation in just one step, the student could have raised each side of the equation to the power \( \frac{3}{2} \).

8. \( 81^{1/4} = 3 \)

9. \( 0^{1/3} = 0 \)

10. \( \sqrt[3]{64} = -4 \)

11. \( \sqrt[12]{25} = 5 \)

12. \( 256^{1/4} = (256)^{1/4} = 4^3 = 64 \)

13. \( 27^{2/3} = \frac{1}{27^{2/3}} = \frac{1}{(27^{2/3})^3} = \frac{1}{3^2} = \frac{1}{9} \)

14. \( (\sqrt[3]{8})^3 = 2 \)

15. \( \frac{1}{(\sqrt[3]{8})^3} = (\sqrt[3]{8})^3 = (-2)^3 = -8 \)

16. \( \sqrt[8]{80} = \sqrt[8]{10} = \sqrt[8]{10} = 2\sqrt[10]{10} \)
Chapter 6, continued

17. \((3^4 \cdot 5^3)^{-1/4} = [(3 \cdot 5)^3]^{-1/4} = (15)^{-1/4} = \frac{1}{\sqrt[4]{15}}\)

18. \((25a^{10}b^{16})^{1/2} = 25^{1/2}(a^{10})^{1/2}(b^{16})^{1/2} = 5a^5b^8\)

19. \[
\sqrt[4]{18x^3y^2} = \frac{\sqrt[4]{18x^3y^2}}{\sqrt[4]{49z^2}} = \frac{3x^2y\sqrt[4]{2}}{7z\sqrt[4]{2}}
\]

20. \(f(x) + g(x) = (4x - 6) + (x + 8) = 5x + 2\)

21. \(f(x) - g(x) = (4x - 6) - (x + 8) = 4x - 6 - x - 8 = 3x - 14\)

22. \(f(x) \cdot g(x) = (4x - 6)(x + 8) = 4x^2 + 26x - 48\)

23. \(f(g(x)) = f(x + 8) = 4(x + 8) - 6 = 4x + 32 - 6 = 4x + 26\)

24. \[
y = \frac{1}{3}x + 4
x = \frac{1}{3}y + 4
x - 4 = \frac{1}{3}y
3x - 12 = y
\]

25. \[
y = 4x^2 + 9, \quad x \geq 0
x = 4y^2 + 9
x - 9 = 4y^2
\frac{1}{4}x - \frac{9}{4} = y^2
\sqrt{\frac{1}{4}x - \frac{9}{4}} = y
\frac{x - 9}{2} = y
\]

26. \[
f(x) = x^3 - 4
y = x^3 - 4
x = y^3
x + 4 = y^3
\sqrt{x + 4} = y
f^{-1}(x) = \sqrt[3]{x + 4}
\]

27. \(y = \sqrt{x} + \sqrt{3} + 5\)

Because \(h = 3\) and \(k = 5\), shift the graph of \(y = \sqrt{x}\) to the left 3 units and up 5 units.

28. \(y = 3\sqrt{x + 1} - 4\)

Because \(h = 1\) and \(k = 4\), shift the graph of \(y = 3\sqrt{x}\) to the left 1 unit and down 4 units.

29. \(y = \sqrt[4]{x} - 4 - 5\)

Because \(h = 4\) and \(k = -5\), shift the graph of \(y = \sqrt[4]{x}\) to the right 4 units and down 5 units.

30. \[
\sqrt{5x - 4} = 2
(\sqrt{5x - 4})^2 = 2^2
5x - 4 = 4
5x = 8
x = \frac{12}{5}
\]

Check: \(
\sqrt{\frac{12}{5} - 4} = \frac{2}{2}
\sqrt{\frac{12}{5} - 4} = \frac{2}{2}
\frac{12}{5} = 2 \quad \checkmark
\)

The solution is \(\frac{12}{5}\).

31. \(3x^{3/4} = 24\)

\[
x^{3/4} = 8
(3^{3/4})^{4/3} = 8^{4/3}
\]

Check: \(3(\frac{16}{3})^{3/4} = 24\)

\[
3(\frac{16}{3})^{3/4} = 24
24 = 24 \quad \checkmark
\]

The solution is 16.
Chapter 6, continued

32. \[ \sqrt{x^2 - 10} = \sqrt{3x} \]
   \[ (\sqrt{x^2 - 10})^2 = (\sqrt{3x})^2 \]
   \[ x^2 - 10 = 3x \]
   \[ x^2 - 3x - 10 = 0 \]
   \[ (x - 5)(x + 2) = 0 \]
   \[ x = 5 \quad \text{or} \quad x = -2 \]

Check \( x = 5 \):

Check \( x = -2 \):

\[ \sqrt{25} - 10 = \sqrt{3(5)} \]
\[ x = \sqrt{3x} \text{ is } x \geq 0 \text{ and } x = -2 < 0, \text{ so } x = -2 \text{ is not a solution.} \]

The solution is 5.

Chapter 6 Test (p. 469)

1. \(-125^{\frac{1}{3}} = -5 \]
2. \(32^{\frac{1}{5}} = 2 \]
3. \(\sqrt[3]{81} = 3 \]
4. \(\sqrt{27} = 3 \]
5. \(8^{\frac{2}{3}} = (8^{\frac{1}{2}})^3 = 2^3 = 8 \]
6. \(16^{-\frac{3}{2}} = \frac{1}{16^{\frac{3}{2}}} = \frac{1}{16^{\frac{1}{2}}^3} = \frac{1}{4^3} = \frac{1}{64} \]
7. \((\sqrt{-27})^2 = (-3)^2 = 9 \]
8. \((\sqrt[4]{64})^4 = \frac{1}{(\sqrt[4]{64})^4} = \frac{1}{4^4} = \frac{1}{256} \]
9. \(\sqrt[3]{88} = \sqrt[3]{8 \cdot 11} = \sqrt[3]{8} \cdot \sqrt[3]{11} = 2\sqrt[3]{11} \]
10. \(\sqrt[6]{16} \cdot \sqrt[3]{8} = \sqrt[6]{128} = \sqrt[6]{32} \cdot 4 = \sqrt[6]{32} \cdot \sqrt[6]{4} = 2\sqrt[6]{4} \]
11. \(\sqrt[12]{1} = \sqrt[12]{\frac{12}{49}} = \frac{2\sqrt[12]{3}}{7} \]
12. \(\sqrt[9]{9} \div \sqrt[9]{9} = \frac{\sqrt[9]{9}}{\sqrt[9]{9}} = \frac{\sqrt[9]{9}}{\sqrt[9]{9}} \cdot \frac{3}{3} = \frac{2\sqrt[9]{9}}{3} \)
13. \(\sqrt[64]{x^3} \cdot x^2 = \sqrt[64]{x^{3 \cdot 2}} = 4x^{\frac{3}{2}} \)
14. \(\sqrt[2]{x^2 \cdot y^2} = \sqrt[2]{x^4 \cdot y^2} = x^2 \cdot y \)
15. \(\sqrt[3]{x^2} \cdot \sqrt[3]{y^2} = \sqrt[3]{x^2} \cdot \sqrt[3]{y^2} = x \cdot y \)
16. \(\sqrt[5]{\frac{25x^3y^6}{36z^5}} = \sqrt[5]{\frac{5x^3y^6}{36z^5}} \cdot \sqrt[5]{\frac{25}{36z^5}} \cdot \sqrt[5]{\frac{36}{36}} = \frac{5x^3y^6}{36z^5} \)

17. \(f(x) + g(x) = (2x + 9) + (3x - 1) = 5x + 8 \)
   The functions \( f \) and \( g \) each have the same domain: all real numbers. So, the domain of \( f + g \) also consists of all real numbers.

18. \(f(x) - g(x) = (2x + 9) - (3x - 1) = 2x + 9 - 3x + 1 = -x + 10 \)
   The functions \( f \) and \( g \) each have the same domain: all real numbers. So, the domain of \( f - g \) also consists of all real numbers.

19. \(f(x) \cdot g(x) = (2x + 9)(3x - 1) = 6x^2 + 25x - 9 \)
   The functions \( f \) and \( g \) each have the same domain: all real numbers. So, the domain of \( f \cdot g \) also consists of all real numbers.

20. \(\frac{f(x)}{g(x)} = \frac{2x + 9}{3x - 1} \)
   The functions \( f \) and \( g \) each have the same domain: all real numbers. Because \( g(\frac{1}{3}) = 0 \), the domain of \( \frac{f}{g} \) is all real numbers except \( x = \frac{1}{3} \).

21. \(f(g(x)) = f(3x - 1) = 2(3x - 1) + 9 \)
   \(= 6x - 2 + 9 = 6x + 7 \)
   The functions \( f \) and \( g \) each have the same domain: all real numbers. So, the domain of \( f(g(x)) \) also consists of all real numbers.

22. \(g(f(x)) = g(2x + 9) = 3(2x + 9) - 1 = 6x + 27 - 1 \)
   \(= 6x + 26 \)
   The functions \( f \) and \( g \) each have the same domain: all real numbers. So, the domain of \( g(f(x)) \) also consists of all real numbers.

23. \(f(f(x)) = f(2x + 9) = 2(2x + 9) + 9 = 4x + 18 + 9 \)
   \(= 4x + 27 \)
   The domain of \( f \) consists of all real numbers. So, the domain of \( f(f(x)) \) also consists of all real numbers.

24. \(g(g(x)) = g(3x - 1) = 3(3x - 1) - 1 = 9x - 3 - 1 \)
   \(= 9x - 4 \)
   The domain of \( g \) consists of all real numbers. So, the domain of \( g(g(x)) \) also consists of all real numbers.

25. \(y = -2x + 5 \)
   \(x = -2y + 5 \)
   \(-2 + 5 = y \)
   \(-\frac{1}{2} + \frac{5}{2} = y \)

26. \(y = \frac{1}{3}x + 4 \)
   \(x = \frac{1}{3}x + 4 \)
   \(x = \frac{1}{3} \)
   \(-3x - 12 = y \)

27. \(f(x) = 5x - 12 \)
   \(y = 5x - 12 \)
   \(x = 5y - 12 \)
   \(x + 12 = 5y \)
   \(\frac{1}{5}x + \frac{12}{5} = y \)
   \(\frac{1}{5}x + \frac{12}{5} = f^{-1}(x) \)

Algebra 2

Worked-Out Solution Key
Chapter 6, continued

28. \[ y = \frac{1}{2}x^2, \ x \geq 0 \]
   \[ x = \frac{1}{2}y^2 \]
   \[ 2x = y^2 \]
   \[ \sqrt{2x} = y \]

29. \[ f(x) = x^3 + 5 \]
   \[ y = x^3 + 5 \]
   \[ x = y^3 + 5 \]
   \[ x - 5 = y^3 \]
   \[ \sqrt[3]{x - 5} = f^{-1}(x) \]

30. \[ f(x) = -2x^3 + 1 \]
   \[ y = -2x^3 + 1 \]
   \[ x = -2y^3 + 1 \]
   \[ x - 1 = -2y^3 \]
   \[ \frac{1}{2}x = y^3 \]
   \[ \sqrt[3]{\frac{1}{2}x} = y \]
   \[ \sqrt[3]{\frac{1}{2}(x - 1)^3} = y \]
   \[ \frac{\sqrt[3]{4} - 4}{\sqrt[3]{8}} = y \]
   \[ \frac{\sqrt[3]{-4x + 4}}{2} = y \]
   \[ \frac{\sqrt[3]{-4x + 4}}{2} = f^{-1}(x) \]

31. \[ y = -6\sqrt{x} \]

<table>
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<tr>
<th>x</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
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<td>6</td>
<td>0</td>
<td>-6</td>
<td>-7.56</td>
</tr>
</tbody>
</table>

The domain and range are all real numbers.

32. \[ y = \sqrt{x - 4} - 2 \]
   Because \( h = 4 \) and \( k = 2 \), shift the graph of \( y = \sqrt{x} \) to the right 4 units and down 2 units.

33. \[ f(x) = -\sqrt[4]{x + 3} + 4 \]
   Because \( h = -3 \) and \( k = 4 \), shift the graph of \( f(x) = -\sqrt{x} \) to the left 3 units and up 4 units.

34. \[ \sqrt[3]{3x} + 7 = 4 \]
   \[ (\sqrt[3]{3x} + 7)^2 = 4^2 \]
   \[ 3x + 7 = 16 \]
   \[ 3x = 9 \]
   \[ x = 3 \]
   Check: \( \sqrt[3]{3} + 7 \neq 4 \)
   \[ \sqrt[16]{16} \neq 4 \]
   \[ 4 = 4 \checkmark \]

35. \[ \sqrt[3]{x} - \sqrt{x + 6} = 0 \]
   \[ \sqrt[3]{x} = \sqrt{x + 6} \]
   \[ (\sqrt[3]{x})^3 = (\sqrt{x + 6})^3 \]
   \[ 3x = x + 6 \]
   \[ 2x = 6 \]
   \[ x = 3 \]
   Check: \( \sqrt[3]{3} - \sqrt{3 + 6} = 0 \)
   \[ \sqrt[9]{9} - \sqrt[9]{9} = 0 \]
   \[ 0 = 0 \]

36. \[ x - 3 = \sqrt{x - 1} \]
   \[ (x - 3)^2 = (\sqrt{x - 1})^2 \]
   \[ x^2 - 6x + 9 = x - 1 \]
   \[ x^2 - 7x + 10 = 0 \]
   \[ (x - 5)(x - 2) = 0 \]
   \[ x = 5 \text{ or } x = 2 \]
   Check \( x = 5 \): \[ 5 - 3 = 2 \checkmark \]
   \[ 2 - 3 = 2 \checkmark \]
   \[ 2 = 2 \checkmark \]
   \[ 2 = 2 \checkmark \]
   \[ 1 = 1 \checkmark \]

   The solution is \( x = 5 \).

37. \[ E = 625s^2 \]
   \[ \frac{\sqrt{E}}{625} = s \]
   \[ \frac{\sqrt{E}}{25} = s \]
Chapter 6, continued

b. When \( E = 120,000 \):
   
   \[
   s = \frac{\sqrt{E}}{25} = \frac{\sqrt{120,000}}{25} = 13.9 \text{ meters per second}
   \]

   \[
   \begin{align*}
   13.9 \text{ m} & \quad 60 \text{ sec} \quad 60 \text{ min} \quad 1 \text{ km} \\
   & \quad 1 \text{ h} \\
   & \quad 1000 \text{ m} = 50 \text{ km} \\
   \end{align*}
   \]
   
   The speed of the car is about 50 kilometers per hour.

c. No; doubling the kinetic energy will increase the speed by a factor of \( \sqrt{2} \).

38. \( h = 0.9(200 - a) \)
   
   \[
   h = 180 - 0.9a
   \]
   
   \[
   a = 200 - \frac{36}{0.9} = 160
   \]
   
   Your average is 160 if your handicap is 36.

Standardized Test Preparation (p. 471)

1. Partial credit
   
   For part a, the composition is formed and simplified correctly.
   
   For part b, the steps are correct for finding \( h(f(20)) \), but the last computation is incorrect.
   
   For part c, the explanation is clear; however, the height at the hip is incorrect from part b.

2. No credit
   
   For part a, the composition is formed incorrectly.
   
   For part b, the student substitutes 10 for \( t \) rather than 20.
   
   For part c, the student’s description is incorrect.

Standardized Test Practice (pp. 472–473)

1. a. \( w = \frac{24}{32}p \)  
   
   \[
   w = \frac{3}{4}p
   \]
   
   \[
   d = \frac{220}{3}p
   \]
   
   \[
   d = 165p
   \]
   
   b. \( d = 220w \)
   
   \[
   d = 220(\frac{3}{4}p)
   \]
   
   \[
   d = 165p
   \]
   
   c. 5th gear: \( w = \frac{24}{19}p \rightarrow d = 220\left(\frac{24}{19}\right)p = 278p \)
   
   10th gear: \( w = \frac{40}{22}p \rightarrow d = 220\left(\frac{40}{22}\right)p = 400p \)
   
   15th gear: \( w = \frac{50}{19}p \rightarrow d = 220\left(\frac{50}{19}\right)p = 579p \)

<table>
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<th>10th</th>
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<td>40</td>
<td>50</td>
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<tr>
<td>Teeth in Freewheel</td>
<td>19</td>
<td>22</td>
<td>19</td>
</tr>
<tr>
<td>Distance per pedal revolution</td>
<td>278 in.</td>
<td>400 in.</td>
<td>579 in.</td>
</tr>
</tbody>
</table>

As the gear increases, the distance traveled per pedal revolution increases.

2. a. 2 people: \( s = 4.62 \sqrt{2} \approx 5 \)
   
   4 people: \( s = 4.62 \sqrt{4} \approx 5.4 \)
   
   8 people: \( s = 4.62 \sqrt{8} \approx 5.8 \)
   
   The speeds of a boat for crews of 2 people, 4 people, and 8 people are about 5 meters per second, 5.4 meters per second, and 5.8 meters per second respectively.

   b. \( \text{distance} = \text{rate} \times \text{time} \)
   
   2 people: \( 2000 = 5t \)
   
   \[
   400 = t
   \]
   
   4 people: \( 2000 = 5.4t \)
   
   \[
   370 = t
   \]
   
   8 people: \( 2000 = 5.8t \)
   
   \[
   344.8 = t
   \]
   
   The speeds of a boat for crews of 2 people, 4 people, and 8 people are about 5 meters per second, 5.4 meters per second, and 5.8 meters per second respectively.

   c. No, doubling the number of rowers does not double the speed; it increased the speed by about 0.4 meter per second.

3. a. \( V = s^3 = 64 \)

   The volume of the cube is 64 cubic inches.

   b. Triangle base: \( V = 0.433ls^2 \)
   
   Hexagon base: \( V = 64 = 1.72ls^3 \)
   
   Octagon base: \( V = 148 = 37l = s^3 \)
   
   The side length is about 5.3 inches.

   c. No; from part (b) the volume was the same for all of the prisms when the prisms had different side lengths.

Algebra 2

Worked-Out Solution Key

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Chapter 6, continued

4. \[ S = 3\pi r^2 \]
   \[ \frac{S}{3\pi} = r^2 \]
   \[ \sqrt[3]{\frac{S}{3\pi}} = r \]

b. \[ V = \frac{2}{3}\pi r^3 = \frac{2}{3}\pi \sqrt[3]{\frac{S}{3\pi}}^3 = \frac{2\pi}{3} \left( \frac{S}{3\pi} \right)^{3/2} = \frac{2\pi}{3} \left( \frac{S}{3\pi} \right)^{3/2} \]
   \[ = \frac{2\pi S}{3 \cdot 3! \sqrt[3]{3\pi}} = \frac{2S}{9\sqrt[3]{3\pi}} \]

c. Volume of smaller snow globe:
   \[ V = \frac{2S}{9\sqrt[3]{3\pi}} \]
   Volume of the larger snow globe:
   \[ V = \frac{2(2S)}{9\sqrt[3]{3\pi}} = \frac{2(2S)}{9\sqrt[3]{3\pi}} \cdot \frac{\sqrt[3]{3\pi}}{\sqrt[3]{3\pi}} = 2\sqrt[3]{2} \left( \frac{2S}{3\pi} \right) \]

Doubling the surface area causes the volume to increase by a factor of \(2\sqrt[3]{2}\).

5. D;
   \[ \sqrt{2x + 3} = 5 \]
   \[ (\sqrt{2x + 3})^2 = 5^2 \]
   \[ 2x + 3 = 25 \]
   \[ 2x = 22 \]
   \[ x = 11 \]

6. C;
   Because the graph of \( y = \sqrt{x} \) is shifted up 3 units, \( h = 0 \) and \( k = 3 \).

7. A;
   \[ y = -2x^3 + 10 \]
   \[ x = -2x^3 + 10 \]
   \[ x - 10 = -2x^3 \]
   \[ 5 - \frac{x}{2} = y^3 \]
   \[ \sqrt[3]{5} - \frac{x}{2} = y \]

8. \( (8x)^{1/5} = 8 \)
   \[ (8x)^{1/5} \cdot 8^{1/5} = 8^{5/5} \]
   \[ 8x = 32 \]
   \[ x = 4 \]

9. \( f(x) = 2x^{1/2}, g(x) = 3x^2 \)
   \[ g(f(x)) = g(2x^{1/2}) = 3(2x^{1/2})^2 = 3(4x) = 12x \]
   \[ g(f(9)) = 12(9) = 108 \]

10. \( \left( \frac{5^5}{2^5} \right)^{1/5} = \left( \frac{5^5}{2^5} \right)^{1/5} = \left( \frac{5^5}{2^5} \right)^{1/5} = \frac{2}{5} \)

11. \( \sqrt[6]{16x^3y^2} = 2x^{1/2}y^{1/2} \)
   \[ 2y^{1/2}x^{1/2} = 2x^{1/2}y^{1/2} \]
   \[ x^{3/4}y^{1/4} = x^{3/4}y^{1/4} \]
   \[ a = \frac{3}{4}, b = \frac{1}{4} \]
   \[ a + b = 1 \]

12. \( \frac{1}{(\sqrt[6]{625})^2} = (\sqrt[6]{625})^2 \]
13. \( f(x) \cdot g(x) = \frac{2}{3x^{0.25}} \cdot 5x^{3.25} = 2x^{-0.25} \cdot 3x^{2.25} = 2x^{3} \)
   When \( x = 3 \):
   \( f(3) \cdot g(3) = 2(3)^3 = 2 \cdot 27 = 54 \)

14. \[ y = \sqrt{x} - 8 + 5 \]
   \[ y = \sqrt[6]{0} - 8 + 5 \]
   \[ y = \sqrt[6]{8} + 5 \]
   \[ y = -2 + 5 \]
   \[ y = 3 \]
   The \( y \)-intercept is 3.

15. \[ f(x) = g(x) \]
   \[ \sqrt[6]{x} - 2 = 3 = \sqrt[6]{x} + 13 \]
   \[ (\sqrt[6]{x} - 2)^2 = (\sqrt[6]{x} + 13)^2 \]
   \[ x - 2 + 6\sqrt[6]{x} - 2 + 9 = x + 13 \]
   \[ 6\sqrt[6]{x} - 2 = 6 \]
   \[ \sqrt[6]{x} - 2 = 1 \]
   \[ (\sqrt[6]{x} - 2)^2 = 1^2 \]
   \[ x - 2 = 1 \]
   \[ x = 3 \]

16. \[ y = 3x + 9 \]
   \[ x = 3y + 9 \]
   \[ x - 9 = 3y \]
   \[ \frac{1}{3}x - 3 = y \]
   The slope of the graph of the inverse is \( \frac{1}{3} \).

17. \( -\sqrt[6]{x} + 4 = \sqrt[6]{x} - 1 + 1 \)
   \[ (-\sqrt[6]{x} + 4)^2 = (\sqrt[6]{x} - 1 + 1)^2 \]
   \[ x + 4 = x - 1 + 2\sqrt[6]{x} - 1 + 1 \]
   \[ 4 = 2\sqrt[6]{x} - 1 \]
   \[ 2 = \sqrt[6]{x} - 1 \]
   \[ 2^2 = (\sqrt[6]{x} - 1)^2 \]
   \[ 4 = x - 1 \]
   \[ 5 = x \]

Check: \( -\sqrt[6]{5} + 4 \neq \sqrt[6]{5} - 1 + 1 \)
\[ \sqrt[6]{9} \neq \sqrt[6]{4} + 1 \]
\[ -3 \neq 3 \]

There is no solution because 5 is extraneous. The results are the same because both methods yield no solution.
Chapter 6, continued

18. \( y = 0.03937x \)
   \[ y = \frac{0.03937}{x} \]
   The inverse function is \( y = 25.4x \).
   From the inverse function, you can convert \( x \) inches to \( y \)
millimeters.
   3 inches: \( w = 25.4(3) = 76 \)
   5 inches: \( A = 25.4(5) = 127 \)
   \( A = \frac{4w}{2} = 127(76) = 9652 \)
   The area of a 3 inch by 5 inch index card is about 9652 square millimeters.

19. Let \( x \) be the original price.
   Function for $10 gift card: \( f(x) = x - 10 \)
   Function for 15% discount: \( g(x) = x - 0.15x = 0.85x \)
   \( g(f(x)) = \) final price when $10 is subtracted before 15% discount is applied.
   \[ g(f(x)) = (x - 10) = 0.85(x - 10) \]
   \( f(g(x)) = \) the final price when 15% discount is applied before the $10 is subtracted.
   \[ f(g(x)) = f(0.85x) = 0.85x - 10 \]
   \[ f(g(27)) = 0.85(27 - 10) = 0.85(17) = $14.45 \]
   \[ f(g(27)) = 0.85(27 - 10) = 22.95 - 10 = $12.95 \]
   \( $14.45 - $12.95 = $1.50 \)
   You can save $1.50 if the store applies the 15% discount first.

20. No, \( f \) and \( g \) are not inverse functions because they are not reflections in the line \( y = x \).

Cumulative Review, Chs. 1–6 (pp. 474–475)

1. (3, 1), \( m = 4 \)
   \[ y - y_1 = m(x - x_1) \]
   \[ y - 1 = 4(x - 3) \]
   \[ y - 1 = 4x - 12 \]
   \[ y = 4x - 11 \]

2. (4, 6), \( m = 7 \)
   \[ y - y_1 = m(x - x_1) \]
   \[ y - 6 = 7(x - 4) \]
   \[ y - 6 = 7x - 28 \]
   \[ y = 7x - 22 \]

3. (3, 2), \( m = -8 \)
   \[ y - y_1 = m(x - x_1) \]
   \[ y - 2 = -8(x - 3) \]
   \[ y - 2 = -8x + 24 \]
   \[ y = -8x + 22 \]

4. (1, -5), \( m = 9 \)
   \[ y - y_1 = m(x - x_1) \]
   \[ y - (-5) = 9(x - 1) \]
   \[ y + 5 = 9x - 9 \]
   \[ y = 9x - 14 \]

5. (-5, 8), \( m = \frac{4}{5} \)
   \[ y - y_1 = m(x - x_1) \]
   \[ y - 8 = \frac{4}{5}(x + 5) \]
   \[ y - 8 = \frac{4}{5}x + 4 \]
   \[ y = \frac{4}{5}x + 12 \]

6. (2, -10), \( m = -\frac{3}{4} \)
   \[ y - y_1 = m(x - x_1) \]
   \[ y - (-10) = -\frac{3}{4}(x - 2) \]
   \[ y + 10 = -\frac{3}{4}x + \frac{3}{2} \]
   \[ y = -\frac{3}{4}x + \frac{17}{2} \]

7. \(-2x + 7 = 15 \)
   \[ -2x = 8 \]
   \[ x = -4 \]
   Check: \(-2(-4) + 7 = 15 \)
   \[ 8 + 7 = 15 \]
   \[ 15 = 15 \checkmark \]

8. \[ |4x - 6| = 14 \]
   \[ 4x - 6 = 14 \quad \text{or} \quad 4x - 6 = -14 \]
   \[ 4x = 20 \quad \text{or} \quad 4x = -8 \]
   \[ x = 5 \quad \text{or} \quad x = -2 \]
   Check \( x = 5 \): \[ |4(5) - 6| = 14 \]
   \[ |20 - 6| = 14 \]
   \[ 14 = 14 \checkmark \]

9. \[ x^2 - 9x + 14 = 0 \]
   \[ (x - 7)(x - 2) = 0 \]
   \[ x - 7 = 0 \quad \text{or} \quad x - 2 = 0 \]
   \[ x = 7 \quad \text{or} \quad x = 2 \]
   Check \( x = 7 \): \[ 7 - 9(7) = 14 \triangleq 0 \]
   \[ 49 - 63 + 14 \triangleq 0 \]
   \[ -14 + 14 \triangleq 0 \]
   \[ 0 = 0 \checkmark \]

10. \[ 4x^2 - 6x + 9 = 0 \]
    \[ x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(4)(9)}}{2(4)} \]
    \[ x = \frac{6 \pm \sqrt{-108}}{-8} \]
    \[ x = \frac{3 \pm 3\sqrt{3}}{4} \]

Algebra 2

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Chapter 6, continued

Check \( \frac{3 - 3\sqrt{3}}{4} \):

\[
4 \left( \frac{3 - 3\sqrt{3}}{4} \right)^2 - 6 \left( \frac{3 - 3\sqrt{3}}{4} \right) + 9 \pm 0
\]

\[
\frac{9 - 18\sqrt{3} + 27}{16} - \frac{18 - 18\sqrt{3}}{4} + 9 \pm 0
\]

\[
\frac{9 - 18\sqrt{3} - 27 - 18 + 18\sqrt{3}}{4} + 9 \pm 0
\]

\[
-\frac{36}{4} + 9 \pm 0
\]

\[
0 = 0 \checkmark
\]

Check \( \frac{3 + 3\sqrt{3}}{4} \):

\[
4 \left( \frac{3 + 3\sqrt{3}}{4} \right)^2 - 6 \left( \frac{3 + 3\sqrt{3}}{4} \right) + 9 \pm 0
\]

\[
\frac{9 + 18\sqrt{3} + 27}{16} - \frac{18 + 18\sqrt{3}}{4} + 9 \pm 0
\]

\[
\frac{9 + 18\sqrt{3} - 27 - 18 - 18\sqrt{3}}{4} + 9 \pm 0
\]

\[
-\frac{36}{4} + 9 \pm 0
\]

\[
0 = 0 \checkmark
\]

11. \( x^3 + 3x^2 - 10x = 0 \)

\( x(x^2 + 3x - 10) = 0 \)

\( x(x + 5)(x - 2) = 0 \)

\( x = 0, x = -5, \) or \( x = 2 \)

Check \( x = 0 \):

\( 0^3 + 3(0)^2 - 10(0) \pm 0 \)

\( 0 = 0 \checkmark \)

Check \( x = -5 \):

\( (-5)^3 + 3(-5)^2 - 10(-5) \pm 0 \)

\( -125 + 75 + 50 \pm 0 \)

\( -50 + 50 \pm 0 \)

\( 0 = 0 \checkmark \)

Check \( x = 2 \):

\( 2^3 + 3(2)^2 - 10(2) \pm 0 \)

\( 8 + 12 - 20 \pm 0 \)

\( 20 - 20 \pm 0 \)

\( 0 = 0 \checkmark \)

12. \( \sqrt{8x + 1} = 7 \)

\( \sqrt{8x + 1} = 7 \)

\( 8x + 1 = 49 \)

\( 8x = 48 \)

\( x = 6 \)

Check: \( \sqrt{8(6) + 1} \pm 7 \)

\( \sqrt{49} \pm 7 \)

\( 7 \pm 7 \checkmark \)
Chapter 6, continued

22. $2x + 5y = 1$
   $3x - 2y = 30$
   $-6x - 15y = -3$
   $6x - 4y = 60$
   $-19y = 57$
   $y = 3$

23. $3x - y = -9$
   $4x + 3y = 14$
   $9x - 3y = -27$
   $4x + 3y = 14$
   $13x = -13$
   $x = -1$

24. $2x + 3y = 47$
   $7x - 8y = -2$
   $2x + 3y = 47$
   $16x + 24y = 376$
   $21x - 24y = -6$
   $37x = 370$
   $x = 10$

25. $(4 - 2i) + (5 + i) = (4 + 5) + (-2 + 1)i = 9 - i$
26. $(3 + 4i) - (7 + 2i) = (3 - 7) + (4 - 2)i = -4 + 2i$
27. $(4 - 2i)(6 + 5i) = 24 + 20i - 12i - 10i^2$
   $= 24 + 8i - 10(-1)$
   $= 34 + 8i$

28. $y = x^2 + 6x + 16$
   $y + 9 = (x^2 + 6x + 9) + 16$
   $y + 9 = (x + 3)^2 + 16$
   $y = (x + 3)^2 + 7$

29. $y = -x^2 + 12x - 46$
   $y - 36 = -(x^2 - 12x + 36) - 46$
   $y - 36 = -(x - 6)^2 - 46$
   $y = -(x - 6)^2 - 10$
30. $y = 2x^2 - 4x + 7$
   $y + 2 = 2(x^2 - 2x + 1) + 7$
   $y + 2 = 2(x - 1)^2 + 7$
   $y = 2(x - 1)^2 + 5$

31. $(2x^2 y)^3 = 2^3 x^6 y^3 = 8x^6 y^3$
32. $(x^4 y^{-3})^2 = x^8 y^{-6} = x^6$
33. $x^4 / x^2 = x^{4-2} = x^2$
34. $x^4 y^{-3} = (x^2 y^{-1/3})^2$
   $= x^{2*2} y^{-2/3}$
   $= x^4 y^{-2/3}$

35. $(x^2 + 11x - 9) + (4x^2 - 5x - 7)$
   $= x^2 + 4x^2 + 11x - 5x - 9 - 7$
   $= 5x^2 + 6x - 16$

36. $(x^3 + 3x - 10) - (2x^2 + x + 8)$
   $= x^3 + 3x - 10 - 2x^2 - 3x - 8$
   $= -3x^3 - 3x^2 - 5x - 10$

37. $(2x^2 + 4x - 7)$
   $= 2(x^2 + 4x - 7)$
   $= 2x^3 + 2x^2 + 8x^2 - 20x - 14x + 35$
   $= 2x^3 + 3x^2 - 34x + 35$

38. $(x^3 - 10x^2 + 33x - 28) / (x - 5)$
   $= 5 - 25 + 40$
   $= 1 - 5 + 8 + 12$

The functions $f$ and $g$ each have the same domain: all real numbers. So, the domain of $f + g$ also consists of all real numbers.
Chapter 6, continued

43. $f(x) \cdot g(x) = (2x - 6)(5x + 1) = 10x^2 - 28x - 6$

The functions $f$ and $g$ each have the same domain: all real numbers. So, the domain of $f \cdot g$ also consists of all real numbers.

44. $f(g(x)) = f(5x + 1) = 2(5x + 1) - 6 = 10x + 2 - 6$

$= 10x - 4$

The functions $f$ and $g$ each have the same domain: all real numbers. So, the domain of $f(g(x))$ also consists of all real numbers.

45. $g(f(x)) = g(2x - 6) = 5(2x - 6) + 1 = 10x - 30 + 1$

$= 10x - 29$

The functions $f$ and $g$ each have the same domain: all real numbers. So, the domain of $g(f(x))$ also consists of all real numbers.

46. $f(x) = 4x + 6$

$g = 4x + 6$

$x = 4y + 6$

$x - 6 = 4y$

$\frac{1}{4}x - \frac{3}{2} = y$

$f^{-1}(x) = \frac{1}{4}x - \frac{3}{2}$

47. $f(x) = \frac{3}{4}x + 7$

$y = \frac{3}{4}x + 7$

$x = \frac{3}{4}y + 7$

$x - 7 = \frac{3}{4}y$

$\frac{7}{3}x - \frac{49}{3} = y$

$f^{-1}(x) = \frac{7}{3}x - \frac{49}{3}$

48. $f(x) = \frac{1}{3}x - \frac{2}{3}$

$y = \frac{1}{3}x - \frac{2}{3}$

$x = \frac{1}{3}y - \frac{2}{3}$

$x + \frac{2}{3} = \frac{1}{3}y$

$3x + 2 = y$

$f^{-1}(x) = 3x + 2$

49. $f(x) = \frac{x^3 - 5}{6}$

$y = \frac{x^3 - 5}{6}$

$x = \frac{y^3 - 5}{6}$

$6x = y^3 - 5$

$6x + 5 = y^3$

$\sqrt[3]{6x + 5} = y$

$f^{-1}(x) = \sqrt[3]{6x + 5}$

50. $f(x) = \sqrt[3]{\frac{2x + 7}{3}}$

$y = \sqrt[3]{\frac{2x + 7}{3}}$

$x = \sqrt[3]{\frac{2y + 7}{3}}$

$x^3 = \frac{2y + 7}{3}$

$3x^3 = 2y + 7$

$3x^3 - 7 = 2y$

$\frac{3x^3 - 7}{2} = y$

$f^{-1}(x) = \frac{3x^3 - 7}{2}$

51. $f(x) = -\frac{8}{9}x^5 + 2$

$y = -\frac{8}{9}x^5 + 2$

$x = -\frac{8}{9}y^5 + 2$

$x - 2 = -\frac{8}{9}y^5$

$\frac{9}{8}x + 9 = y^5$

$\sqrt[5]{\frac{9}{8}x + 9} = y$

$f^{-1}(x) = \sqrt[5]{\frac{9}{8}x + 9}$

52. Cost of bicycle = Amount saved per week \cdot Number of weeks

\[
\begin{align*}
360 &= 30 \cdot x \\
360 &= 30x \\
12 &= x
\end{align*}
\]

It will take 12 weeks to save enough money to buy the bicycle.
Chapter 6, continued

53. Charitable Donations

There are 3500 lower-level seats and 6300 upper-level seats sold for the ice show.

\[(x_1, y_1) = (2, 2.3); (x_2, y_2) = (6, 4.3)\]

\[m = \frac{4.3 - 2.3}{6 - 2} = \frac{2}{4} = \frac{1}{2}\]

\[y - y_1 = m(x - x_1)\]

\[y - 2.3 = \frac{1}{2}(x - 2)\]

\[y - 2.3 = \frac{1}{2}x - 1\]

\[y = \frac{1}{2}x + 1.3\]

When \(x = 8\): \[y = \frac{1}{2}(8) + 1.3 = 5.3\]

The income for ticket sales for Friday and Saturday nights’ concerts was $4642.

56. \[h = -16t^2 + h_0\] → \[h = -16t^2 + 85\]

When \(h = 0: 0 = -16t^2 + 85\)

\[-85 = -16t^2\]

\[5.3 = t^2\]

\[\pm\sqrt{5.3} = t\]

\[\pm 2.3 = t\]

Reject the negative solution, \(-2.3\). The rock will hit the ground in about 2.3 seconds.

57. (0, 6), (20, 56), (36, 24)

\[y = ax^2 + bx + c\]

\[6 = a(0)^2 + b(0) + c\] → \[6 = c\]

\[56 = a(20)^2 + b(20) + c\] → \[56 = 400a + 20b + c\]

\[24 = a(36)^2 + b(36) + c\] → \[24 = 1296a + 36b + c\]

\[400a + 20b + 6 = 56\] → \[400a + 20b = 50\]

\[1296a + 36b + 6 = 24\] → \[1296a + 36b = 18\]

\[400a + 20b = 50\]

\[3600a + 180b = 450\]

\[-6480a - 180b = -90\]

\[-2880a = 360\]

\[a = \frac{1}{8}\]

\[400\left(-\frac{1}{8}\right) + 20b = 50\]

\[-50 + 20b = 50\]

\[20b = 100\]

\[b = 5\]

The solution is \(a = \frac{1}{8}\), \(b = 5\), and \(c = 6\). A quadratic function that models the baseball’s path is

\[y = \frac{1}{8}x^2 + 5x + 6\]

Algebra 2

Worked-Out Solution Key

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58. Volume = \( V = \ell \cdot w \cdot h \)
\[ 200 = (x)(x)(x + 3) \]
\[ 200 = x^3(x + 3) \]
\[ 0 = x^3 + 3x^2 - 200 \]
Possible rational zeros: \( \pm 2, \pm 4, \pm 5, \pm 8, \pm 10, \pm 20, \pm 25, \pm 40, \pm 50, \pm 100, \pm 200 \).
\[
\begin{array}{cccc}
5 & 1 & 3 & 0 -200 \\
5 & 40 & 200 \\
1 & 8 & 40 & 0
\end{array}
\]
\[ x - 5 = 0 \text{ or } x^2 + 8x + 40 = 0 \]
\[ x = 5 \text{ No real solution } \]
\[ x^3 + 3x^2 - 200 = (x - 5)(x^2 + 8x + 40) \]
The dimensions of a mold are 5 inches by 5 inches by 8 inches.

59. The third-order differences are constant, so the data can be represented by a cubic function. Using the cubic regression feature on a graphing calculator, a model is
\[ y = x^3 - 5x + 6. \]
When \( x = 9; y = 9^3 - 5(9) + 6 = 690 \)
The profit in the ninth month will be $690.

60. \[ V = \frac{4}{3} \pi r^3 \]
\[ 7240 = \frac{4}{3} \pi r^3 \]
\[ 5430 = \pi r^3 \]
\[ \frac{5430}{\pi} = r^3 \]
\[ \sqrt[3]{5430} = r \]
\[ 12 \approx r \]
The radius of the beach ball is about 12 inches.